Policy Portfolio for Banks: Deposit Insurance and Liquidity Injection

Chong Huang   Lin Shen   Junyuan Zou *

August 30, 2024

Abstract

Banking crises pose significant threats to our economy, leading to the implementation of policy measures such as deposit insurance and liquidity injection to strengthen financial stability and optimize resource allocation efficiency. This paper investigates the dynamic interplay between deposit insurance and liquidity injection. Facing uncertainty regarding bank health and depositor liquidity shocks, policymakers decide liquidity injection based on withdrawals. While higher deposit insurance coverage can mitigate panic runs, it may undermine the effectiveness of liquidity injections. We demonstrate that liquidity injection overshadows deposit insurance. Consequently, the optimal policy portfolio entails zero deposit insurance, enhancing resource allocation efficiency but leading to more panic runs.

*Chong Huang, Paul Merage School of Business, University of California, Irvine, chong.h@uci.edu; Lin Shen, INSEAD, lin.shen@insead.edu; Junyuan Zou, INSEAD, junyuan.zou@insead.edu. For very helpful comments, we thank Elena Carletti, Itay Goldstein, Alexander Guembel, Xuewen Liu, Ming Yang, Liyan Yang, and conference and seminar participants at University of Hong Kong, University College London, University of Cambridge and INSEAD Symposium. We are grateful for the support of ABN AMRO Research Fund.

1 Introduction

Banking crises have resulted in prolonged economic downturns and have had adverse effects on overall welfare (Laeven and Valencia 2013; Laeven and Valencia 2020). What's more concerning is their enduring presence. For instance, in March 2023, Silicon Valley Bank (SVB) faced insolvency following a bank run, marking the second-largest bank failure in U.S. history and significantly disrupting financial services for many leading technology companies. Subsequently, First Republic Bank (FRB) also experienced substantial withdrawals, leading to its closure and acquisition by JPMorgan Chase.

Given the significant adverse consequences caused by bank failure, policy interventions become imperative. Indeed, to prevent bank runs, many countries have long adopted deposit insurance policies. For example, the U.S. implemented federal deposit insurance after the Great Depression, and the insured limit per account is now $\$250,000$. However, the recent bank failure highlights that deposit insurance does not always suffice, which has led to the broader adoption of alternative policy tools, with liquidity injection emerging as a prominent strategy (Laeven and Valencia 2020). For example, in the U.S., the Federal Reserve operates the 'discount window,' allowing banks to access liquidity to address unforeseen funding needs.[1]

Examined individually, both deposit insurance and liquidity injection contribute to bank stability and enhance resource allocation efficiency in the short term, albeit through distinct mechanisms.[2] Deposit insurance acts as a proactive measure, reducing depositors' incentives to withdraw and mitigating the adverse effects of bank maturity mismatch, thus safeguarding bank stability and averting asset liquidation during panic runs. Conversely, liquidity injection serves as a reactive solution, addressing immediate liquidity needs to preserve the integrity of a bank's high-quality long-term investments. Anticipating this intervention, depositors are less inclined to withdraw, reinforcing bank stability.

However, when integrated into a policy portfolio, deposit insurance and liquidity injection exhibit dynamic interactions. The introduction of deposit insurance influences depositors' behavior and, consequently, the liquidity conditions of banks, affecting subsequent liquidity

[1]Following the failure of SVB, the Federal Reserve also introduced the 'Bank Term Funding Program,' which serves as an additional liquidity source to alleviate the pressure on banks to hastily sell high-quality securities during times of stress. Such a program, however, has ceased making new loans since March 2024.

[2]In the long run, both policy instruments may lead to moral hazard problems that hurt both financial stability and resource allocation efficiency. See, for example, Allen, Carletti, Goldstein, and Leonelloe (2018) for the moral hazard caused by government guarantee, and Farhi and Tirole (2012), Nosal and Ordoñez (2016), Keister (2016), and Keister and Narasiman (2016) for the moral hazard caused by government bailout.

injections. Likewise, the anticipation of liquidity injection may alter depositors' incentives, thereby influencing the effectiveness of deposit insurance. Does a combination of these two policies generate any synergistic effects? Or, will one substitute the other? For a policymaker striving to optimize social welfare, what constitute the optimal policy portfolio?

This paper addresses these questions. We develop a model based on Goldstein and Pauzner (2005). In our model, the policymaker aims to maximize social welfare. She initially commits to a deposit insurance scheme, ensuring a minimum payment to all depositors in the event that the bank fails to meet their obligations as outlined in demand deposit contracts. Depositors may encounter liquidity shocks and so have to withdraw from the bank. Those depositors who are not facing liquidity shocks (and are called *patient depositors*) observe heterogeneous private signals about the bank fundamentals and make withdrawal decisions simultaneously. The policymaker, upon observing withdrawal size, determines the amount of liquidity injection necessary to cover all or part of these withdrawals. Subsequently, the bank may proceed with its investments, provided that it retains a positive balance of long-term assets after fulfilling withdrawal obligations.

We show that the two policy instruments do not exhibit positive synergy in our model. In equilibrium, to optimize social welfare, the policymaker refrains from committing to any deposit insurance but instead injects liquidity into the bank only when the withdrawal size is small. This strategy entails permitting more panic runs, thereby enabling the policymaker to utilize ex-post liquidity injection to enhance resource allocation efficiency.

We begin our analysis with deposit insurance in isolation, considering a scenario where the policymaker commits to injecting zero liquidity – a situation akin to that discussed by Allen, Carletti, Goldstein, and Leonelloe (2018). We demonstrate the beneficial impact of deposit insurance on financial stability and resource allocation. In equilibrium, the policymaker commits to a positive deposit insurance limit, significantly reducing depositors' incentives for withdrawals and mitigating panic runs. Specifically, as depositors' private signal noise diminishes, withdrawals occur only when their signals fall below a threshold that indicates a zero NPV of the bank's investment – a threshold considerably lower than under laissez-faire conditions. Consequently, compared to scenarios without deposit insurance, a larger number of viable banks experience fewer withdrawals, underscoring how deposit insurance improves resource allocation by preventing excessive liquidation of long-term investments with positive NPVs.

On the other hand, intuitively, deposit insurance may incur costs on resource allocation efficiency. In cases where banks hold long-term investments with negative NPVs, it is optimal to liquidate a larger portion of their investments. However, deposit insurance diminishes depositors' incentives to withdraw, reducing monitoring of the bank's investments and resulting in a social loss.[3]

We then explore our core model in which the policymaker is unable to commit to refraining from liquidity injection. (The analysis of liquidity injection in isolation is a special case of our core model, wherein the policymaker commits to zero deposit insurance.) Despite the well-known complexity of global-game-based bank run models, particularly when incorporating policy interventions at different time points, we successfully analyze the policymaker's policy portfolio choice within a tractable model and unveil some unexpected findings.

The first challenge in solving the model pertains to the policymaker's decision on liquidity injection, compounded by uncertainty regarding the origins of bank runs. Discerning whether such runs are solely driven by depositors' liquidity shocks, indicative of a mere liquidity crisis, or if they signal broader depositor confidence erosion, hinting at underlying bank issues – especially regarding bank investment quality – is a formidable task for policymakers, particularly given the constrained timeframe within which decisions must be made.

In our model, the policymaker derives insights into bank fundamentals based on withdrawal size. We show that, given depositors' strategies, greater withdrawals signify weaker bank fundamentals. Essentially, a larger volume of withdrawals suggests the presence of more patient depositors who perceive adverse signals regarding bank fundamentals. Consequently, in equilibrium, the policymaker opts for full liquidity injection to prevent any liquidation of long-term investments when the bank experiences a small amount of withdrawals. However, if withdrawals exceed a certain threshold, the policymaker refrains from injecting any liquidity, as it indicates the bank's investment has a negative NPV.

The depositors' withdrawal strategies influence the policymaker's liquidity injection threshold. In equilibrium, depositors employ a symmetric cutoff strategy, wherein each patient depositor withdraws if and only if their private signal falls below a cutoff. Consequently, with the same amount of withdrawals, a lower withdraw cutoff implies a higher proportion of patient depositors seeking withdrawal, signaling weaker bank fundamentals and discouraging the policymaker to inject liquidity. Hence, as the depositors' withdrawal threshold rises, so does the policymaker's liquidity injection cutoff.

The uncertainty surrounding bank fundamentals suggests the inefficiencies of liquidity in-

[3] In a recent study by Martin, Puri, and Ufier (2023), it was found that depositors continued to deposit funds into a bank nearing failure, up to the deposit insurance limit. The influx of newly insured deposits effectively mitigated the bank's losses from uninsured deposits, allowing the bank to keep more low-quality investments and thus highlighting the social cost associated with deposit insurance.

jection in improving resource allocation. Instances arise where despite sound bank fundamentals, the policymaker refrains from injecting liquidity after observing significant withdrawals due to substantial liquidity shocks. Conversely, there are cases where, despite poor bank fundamentals, the policymaker opts for full liquidity injection following a small amount of withdrawals. However, the latter scenario is less worrisome, as the small amount of withdrawals indicates that the bank's fundamentals cannot be extremely bad.

Depositors take into consideration the policymaker's liquidity injection strategy when making their withdrawal decisions. Intuitively, a higher liquidity injection threshold increases the likelihood of the bank surviving runs, thereby enhancing the payoff of staying with the bank. Consequently, in equilibrium, a higher liquidity injection threshold results in a lower withdrawal cutoff. This optimal response underscores the positive impact of liquidity injection on financial stability: With the anticipation of liquidity injection, patient depositors are less inclined to withdraw, effectively mitigating panic runs.

Finally, we determine the policymaker's optimal deposit insurance, which presents a second challenge in solving the model. This challenge arises from the interplay between deposit insurance effects, which influence both depositor withdrawal strategies and liquidity injection decisions. There is, unfortunately, no explicit solution for the continuation play following each deposit insurance limit. We develop an approach to conquer such a challenge. Instead of analyzing the effect of a deposit insurance limit on social welfare, we can study the effect of the depositors' withdrawal threshold on social welfare. This is because in equilibrium, one deposit insurance limit determines a unique withdrawal threshold.

Consider a higher depositor withdrawal threshold. We first show that it has a positive impact on resource allocation efficiency on the intensive margin. Given any bank fundamentals, a larger proportion of patient depositors will withdraw, causing the bank to liquidate more long-term investments in the absence of liquidity injection. This is efficient when bank fundamentals are poor – with negative NPV investments, larger liquidation of long-term investments enhances resource allocation efficiency. Conversely, when bank fundamentals are strong, liquidating long-term investments becomes inefficient. However, the former effect outweighs the latter one for two reasons. On one hand, in instances of strong bank fundamentals, it is more likely that the amount of total withdrawals is small, prompting the policymaker to inject full liquidity to preserve the bank's long-term investments. On the other hand, if liquidity shocks to depositors exceed the policymaker's liquidity injection threshold, panic runs will have negligible impact on resource allocation efficiency. Therefore, on the intensive margin, a lower withdrawal threshold positively influences resource allocation efficiency.

Second, we find that a higher depositor withdrawal threshold positively influences resource allocation on the extensive margin. Let us consider a scenario where the policymaker behaves irrationally and fails to adjust her belief system despite the higher depositor withdrawal threshold. In this case, a greater number of patient depositors withdraw across all bank fundamentals, leading the policymaker to inject liquidity only in instances of smaller liquidity shocks. Consequently, many banks receiving full liquidity support receive zero liquidity injection from the policymaker following the increase in the depositor withdrawal threshold. This change in liquidity injection is more pronounced for banks with weak fundamentals, as they require smaller liquidity shocks to obtain liquidity support, thereby magnifying the impact of patient depositor decisions on the policymaker's liquidity injection strategy (because the measure of patient depositors is larger). Thus, the rise in depositor withdrawal thresholds prevents more weak banks (compared to strong banks) from receiving liquidity injections, thereby enhancing resource allocation efficiency.

A third effect of a higher depositor withdrawal threshold on resource allocation efficiency is that the policymaker adjusts her liquidity injection cutoff as a best response. However, this effect is negligible, as the marginal impact of liquidity injection cutoff on social welfare is zero at the optimal liquidity injection strategy. This result is a technical application of the Envelope Theorem.

Combining the three effects of a higher depositor withdrawal threshold on the resource allocation efficiency, we find that liquidity injection *overshadows* deposit insurance. Consequently, contrary to conventional wisdom, the policymaker opts for a zero deposit insurance limit in equilibrium. That is, to optimize resource allocation efficiency, the policymaker strategically allows more panic bank runs and thus sacrifices financial stability.

Related literature Our paper is related to a large literature on bank fragilities due to strategic complementarities among depositors and panic-driven runs (Diamond and Dybvig 1983). We follow Rochet and Vives (2004) and Goldstein and Pauzner (2005) to analyze bank runs in a global-games framework, which is useful for policy analysis because it pins down a unique equilibrium and links it to bank fundamentals. Also closely related, Chari and Jagannathan (1988) model withdrawals by uninformed depositors who cannot disentangle withdrawals due to liquidity need and poor bank fundamentals. We focus on policy analysis, and in our model, it is the policymaker who cannot disentangle withdrawals due to aggregate liquidity need and panic runs.

This literature also explores policies that reduce panic-driven runs and enhance the stability of the banking system. A leading policy is deposit insurance. Diamond and Dybvig (1983) provides a rationale for deposit insurance in preventing panic-driven bank runs. Despite its stabilizing effect, deposit insurance has been criticized for moral hazard (Keeley 1990; Cooper and Ross 2002). More recently, Allen, Carletti, Goldstein, and Leonelloe (2018) conduct thorough analysis to compare various forms of deposit insurance and show that it is welfare-improving despite the moral hazard problem. Deposit insurance also faces challenges when the aggregate liquidity need is uncertain (Wallace 1988; Chari 1989). Our paper also features aggregate liquidity need. We contribute to the literature by examining deposit insurance in conjunction with ex post liquidity injection. This uncovers the interaction between the two policies such that the stabilizing effect of deposit insurance can be completely overshadowed by liquidity injection. Dávila and Goldstein (2023) take the approach of sufficient statistics to guide policymakers in deciding the optimal deposit insurance coverage. Our paper calls for a holistic approach to assess the optimal deposit insurance coverage in a policy portfolio.

Another popular policy is ex post bailout. The literature sometimes conflates bailouts with deposit insurance as two forms of government guarantee. However, they differ in timing: bailouts are reactive measures taken after a crisis, while deposit insurance is a proactive commitment. Along this line, the literature has emphasized the inefficiency of ex post bailout due to the distortion of ex ante incentives (Holmström and Tirole 1998; Farhi and Tirole 2012; Keister 2016; Keister and Narasiman 2016; Chari and Kehoe 2016). Nosal and Ordoñez (2016) argues that uncertainties about the nature of distress can serve as a self-disciplinary device to mitigate the lack of commitment for ex post bailouts. Cooper and Kempf (2016) consider a policy portfolio of ex post deposit insurance and orderly resolution. Mitkov (2020) factors in wealth inequality in the optimal ex post bailout policy. Our paper contributes to this literature by highlighting the advantage of bailout policy in its flexibility in providing contingent liquidity support to banks in various sizes, which gives rise to its dominance over ex ante deposit insurance when both policy tools are allowed.

Finally, our paper contributes to the literature on government learning from the economic activities of other agents. Bond and Goldstein (2015) analyze an economic setting where the government learns from the financial market that partially aggregates investors' private information.[4] Ahnert, Machado, and Pereira (2023) explore how the government learns from the

[4]More broadly, our paper is related to the literature on informational feedback between real decisions and financial markets. See Bond, Edmans, and Goldstein (2012) and Goldstein (2023) for excellent reviews of this literature. However, in most papers in this literature, due to asset price, investors' behavior exhibits strategic substitution.

financial market when an investor possesses market power. Our model differs from these studies by focusing on depositors who have strategic complementary incentives in their withdrawal decisions. Goldstein, Ozdenoren, and Yuan (2011) investigate how a central bank learns from speculative currency attacks to decide whether to maintain a currency peg, and their setting does feature strategic complementarities among speculators. Relative to this paper, our novel contribution lies in showing that the policymaker's proactive policy — deposit insurance — affects depositors' withdrawal incentives, thereby influencing the information available to guide the policymaker's reactive policy — liquidity injection.

2 A Model for Policy Portfolio

This section develops a model for policy portfolio. The economy is populated with one bank, one policymaker, a continuum $[0, 1]$ of depositors. All agents are risk-neutral. There are three periods indexed by $t = 0, 1, 2$, and no time discounting.

The Bank There is one bank in our model. At $t = 0$, the bank collects one unit of capital from a unit mass of depositors in the form of demandable debt, specified below. The bank has access to a long-term investment technology. Each unit of long-term investment generates a gross return of 1 if liquidated at $t = 1$. If held until $t = 2$, each unit of investment yields a gross return of $R > 1$ with probability $P(\theta)$ and 0 with probability $1 - P(\theta)$ at $t = 2$. Here, θ represents the fundamentals of the bank. Without loss of generality, we assume that θ follows a standard normal distribution, with a cumulative distribution function $\Phi(\cdot)$ and a probability density function $\phi(\cdot)$. The probability function $P(\theta)$ is differentiable and strictly increasing, $\lim_{\theta \to -\infty} P(\theta) = 0$, and $\lim_{\theta \to +\infty} P(\theta) = 1$.[5]

At $t = 1$, depositors can withdraw their deposits, and the bank must liquidate part or all of its long-term investment to fulfill the withdrawal requests. For simplicity, we assume that the bank liquidates its long-term investment only if it is necessary to fulfill the withdrawals requested at $t = 1$.

Demand Deposit Contract The bank signs a demand deposit contract, (r_1, r_2), with each depositor at $t = 0$. To focus on the interplay between deposit insurance and liquidity injection, we assume that the demand deposit contract is exogenously given. This assumption can also be

[5] Note that it is equivalent to an environment where bank fundamentals follow a general normal distribution with mean μ and precision γ, and the probability function takes a form of $P(\mu + (\sqrt{\gamma})^{-1}\theta)$.

justified by the fact that banks are price takers, so the demand deposit contract is the prevailing one in the whole banking industry. We assume that $R > r_2 > r_1 > 1$.

According to such a contract, if a depositor requests to withdraw at $t = 1$, the bank promises to pay him $r_1 > 1$ provided that it has sufficient funds. Similarly, if the depositor waits until $t = 2$, the bank promises to pay him up to r_2 subject to the availability of funds. At $t = 2$, after repaying depositors, the bank's equity holder consumes the residual funds, if any.

Depositors There is a continuum of depositors, uniformly distributed over $[0, 1]$. At $t = 0$, each depositor i deposits one unit of capital into the bank. At $t = 1$, each depositor i may experience an individual liquidity shock with probability λ. By the Law of Large Numbers, a measure λ of depositors will become impatient. Therefore, λ represents the aggregate liquidity needs and is uniformly distributed over $[0, 1]$. λ is realized at the beginning of $t = 1$. For simplicity, we assume that depositors have no information about the realization of λ, but each depositor observes a private signal s_i about the bank's fundamental θ,

$$s_i = \theta + \xi_i, \tag{1}$$

where $\xi_i \sim \mathcal{N}\left(0, \beta^{-1}\right)$.

If a depositor experiences a liquidity shock, he becomes impatient and can only consume at $t = 1$. A depositor without a liquidity shock remains patient and can consume at both $t = 1$ and $t = 2$. Then, any depositor i's ex-post payoff is

$$u_i = \begin{cases} c_1, & \text{if depositor } i \text{ is impatient;} \\ c_1 + c_2, & \text{if depositor } i \text{ is patient.} \end{cases} \tag{2}$$

As a result, all impatient depositors optimally demand early withdrawal at $t = 1$. The patient depositors, by contrast, choose between withdrawing at $t = 1$ or staying until $t = 2$ based on their private signals.

Policymaker and Policy Portfolio The policymaker considers a *policy portfolio* consisting of two tools: deposit insurance and liquidity injection. Since the deposit insurance is a commitment by policymaker before depositors make their withdrawal decisions, while the liquidity injection is provided afterwards, we say deposit insurance is an *ex-ante policy tool*, while liquidity injection is an *ex-post policy tool*.

Specifically, at $t = 0$, the policymaker commits to a deposit insurance $K \in [0, r_1]$, which guarantees a minimum cash flow of K to all depositors regardless of their withdrawal decisions. Therefore, whenever the bank is in short of funds to repay depositors at least K, the policymaker will transfer money to the bank to guarantee each remaining depositor a repayment K.

At $t = 1$, the policymaker observes the total withdrawal demand, denoted as $m \in [0, 1]$. Then, she decides the amount of liquidity injection $\ell \in [0, mr_1]$ into the bank. Such liquidity injection can be used to repay depositors who request to withdraw at $t = 1$ only. We set the constraint of liquidity injection so that the policymaker cannot use the liquidity injection to invest in the bank's project (i.e., $\ell > mr_1$), nor can she liquidate the bank's investment (i.e., $\ell < 0$). We assume that the policymaker refrains from injecting liquidity when she is indifferent.

Payoffs The active players in our model are the depositors and the policymaker. The bank and its equity holder are both passive.

Denote by $d_i = 1$ the decision of depositor i to request to withdraw at $t = 1$ and by $d_i = 0$ the decision to stay with the bank until $t = 2$. Then, depositor i's ex-post payoff at $t = 1$ depends on his withdrawal decision d_i, the measure of day-1 withdrawals m, the liquidity injection made by the policymaker ℓ, and the deposit insurance K. Specifically, if depositor i requests to withdraw at $t = 1$, his payoff is

$$u_i(d_i = 1) = \begin{cases} r_1, & \text{if } \frac{1+\ell}{m} \geq r_1; \\ \frac{1+\ell}{m}, & \text{if } r_1 > \frac{1+\ell}{m} > K; \\ K, & \text{if } K \geq \frac{1+\ell}{m}. \end{cases} \tag{3}$$

On the other hand, if depositor i does not request to withdraw until $t = 2$, his expected payoff at day 1 is

$$u_i(d_i = 0) = \begin{cases} P(\theta) r_2 + (1 - P(\theta)) K, & \text{if } \frac{1+\ell}{m} \geq r_1 \text{ and } \frac{1+\ell-mr_1}{1-m} R \geq r_2; \\ P(\theta) \frac{1+\ell-mr_1}{1-m} R + (1 - P(\theta)) K, & \text{if } \frac{1+\ell}{m} \geq r_1 \text{ and } r_2 > \frac{1+\ell-mr_1}{1-m} R > K; \\ K, & \text{if } \frac{1+\ell}{m} \geq r_1 \text{ and } K \geq \frac{1+\ell-mr_1}{1-m} R; \\ K, & \text{if } r_1 > \frac{1+\ell}{1-m} > K; \\ K, & \text{if } K \geq \frac{1+\ell}{1-m}. \end{cases} \tag{4}$$

The policymaker is risk neutral, and aims to maximize the welfare of the economy — the total payoffs of all depositors' and the equity holder minus the cost of transfers from the policymaker to the bank or its depositors. We assume that such transfers, either in the form of liquidity injection or deposit insurance payments, have a marginal cost of 1. Therefore, we can express the policymaker's payoff at $t = 1$ when she makes the liquidity injection decision as follows,[6]

$$u_g = \max\{1 - (mr_1 - \ell), 0\}(\mathbb{E}[P(\theta)|m]R - 1). \tag{5}$$

Note that the policymaker ultimately cares about real efficiency, and her payoff u_g is simply the total NPV of the bank's long-term investment held until maturity. Specifically, $mr_1 - \ell$ is the bank's shortage of liquidity to fulfill the early withdrawal demand of its depositors. This liquidity gap must be covered by liquidating its long-term investment prematurally. Hence, $1 - (mr_1 - \ell)$ represents the amount of long-term investment held until maturity at $t = 2$, if it is positive. If $1 - (mr_1 - \ell) \leq 0$, the bank is unable to meet all early withdrawal demand even though it has liquidated all long-term investment, thus it fails.

It is also worth noting that the deposit insurance limit K does not enter the welfare expression (Equation (5)) directly. This is because deposit insurance payments are simply transfers from the policymaker to the depositors, and the increase in depositors' payoffs will cancel out with the cost of transfer. However, deposit insurance influences the run incentive of patient depositors and thus affects welfare indirectly through m. In contrast, liquidity injection reduces premature liquidation of long-term investment and therefore shows up in the expression directly. In addition, as we illustrate in section 4.2, liquidity injection also indirectly affects welfare through m because depositors become less inclined to run in anticipation of liquidity injection.

Strategy and Equilibrium The timeline of our model is shown in Figure 1. At $t = 0$, the policymaker commits to a deposit insurance. At $t = 1$, upon observing heterogeneous private signals, all depositors simultaneously decide to withdraw from the bank or to keep their deposits in the bank until $t = 2$. The policymaker observes the measure of withdrawal requests and decides the amount of liquidity to inject into the bank. At $t = 2$, the bank's investment return, as well as payoffs are realized.

The policymaker's behavior strategy at $t = 0$ is a choice of deposit insurance $K \in [0, r_1]$. Any depositor i's strategy is a mapping $D_i : [0, r_1] \times \mathbb{R} \to \{0, 1\}$; that is, for any deposit insurance K and any private signal $s_i \in \mathbb{R}$, depositor i decides whether to withdraw at $t = 1$. The policymaker's behavior strategy at $t = 1$ is a mapping $\mathcal{L} : [0, r_1] \times [0, 1] \to [0, mr_1]$; that is, for any committed deposit insurance K and any measure of early withdrawals $m \in [0, 1]$, the policymaker chooses $\ell \in [0, mr_1]$ to inject into the bank.

We are interested in a *monotone perfect Bayesian equilibrium*.

[6] To simplify the expression, we subtract a constant 1, the initial capital, from the welfare.

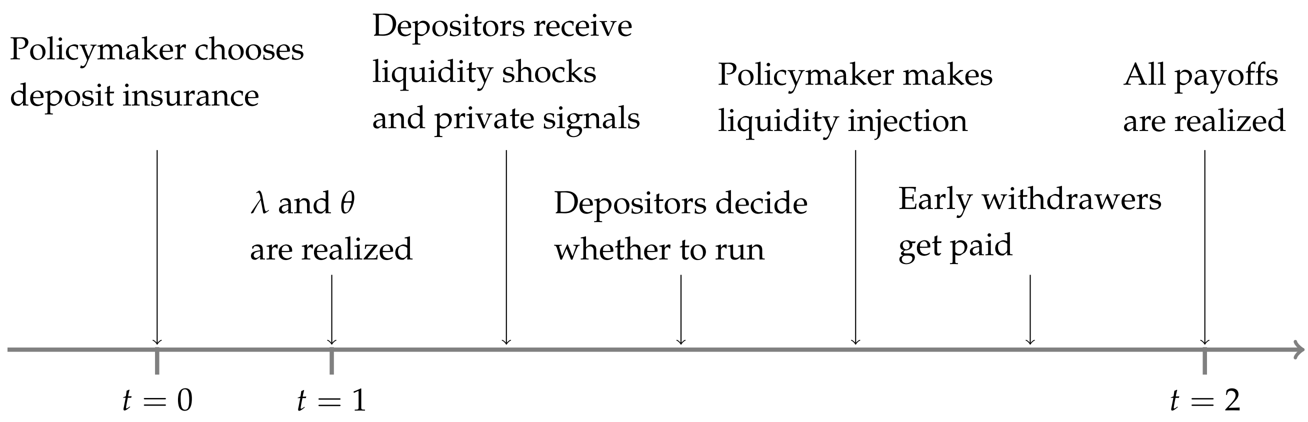


Figure 1: **Timing**

Definition 1 *The policymaker's deposit insurance K^*, her liquidity injection strategy $\mathcal{L}^*$, and the depositors' strategy D_i^* for all $i \in [0,1]$ constitute a monotone perfect Bayesian equilibrium if*

1. *the deposit insurance K^* maximizes the policymaker's expected payoff at $t = 0$, $\mathbb{E}\left(u_g\right)$, given the depositors' strategies $\{D_i^*\}_{i\in[0,1]}$ and her liquidity injection strategy $\mathcal{L}^*$;*

2. *given any deposit insurance K, other depositors' strategies $\{D_j^*\}_{j\neq i}$, and the policymaker's liquidity injection strategy $\mathcal{L}^*$, any depositor i's withdrawal choice at $t = 1$, $D_i^*(K, s_i)$, maximizes her expected payoff at $t = 1$;*

3. *given the depositors' strategies $\{D_j^*\}_{i\in[0,1]}$, for any deposit insurance K and any measure of early withdrawals m, the liquidity injection $\ell = \mathcal{L}^*\left(K, m\right)$ maximizes the policymaker's payoff u_g; and*

4. *all players update their beliefs according to Bayes' rule.*

3 Benchmark: Deposit Insurance Alone

In this section, we explore a benchmark case where the policymaker utilizes deposit insurance alone and commits to inject zero liquidity. We solve for the equilibrium by backward induction in two steps. Section 3.1 investigates the equilibrium run strategy in response to a deposit insurance policy. Section 3.2 characterizes the optimal deposit insurance policy. We will compare this benchmark case with the optimal policy portfolio in Section 4.

3.1 Panic Bank Run

Given a deposit insurance policy $K \in [0, r_1]$, we can express any depositor i's payoff from withdrawing at $t = 1$, conditional on the measure of withdrawals m and bank fundamentals θ, as

$$c_1^{DI}(m, \theta; K) = \begin{cases} K, & \text{if } m \geq K^{-1}; \\ \frac{1}{m}, & \text{if } K^{-1} > m \geq r_1^{-1}; \\ r_1, & \text{otherwise.} \end{cases} \tag{6}$$

Similarly, depositor i's payoff from staying with the bank until $t = 2$ can be expressed as

$$c_2^{DI}(m, \theta; K) = \begin{cases} K, & \text{if } m \geq \frac{R-K}{r_1 R - K}; \\ p(\theta)\frac{(1-mr_1)R}{1-m} + (1-p(\theta))K, & \text{if } \frac{R-K}{r_1 R-K} > m \geq \frac{R-r_2}{r_1 R - r_2}; \\ p(\theta) r_2 + (1-p(\theta))K, & \text{otherwise.} \end{cases} \tag{7}$$

To understand depositors' run incentives, it is useful to examine their incremental payoff from staying:

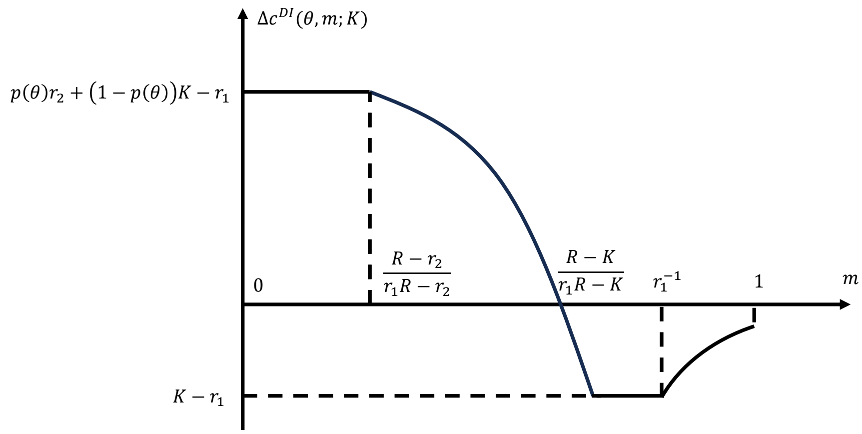
$$\Delta c^{DI}(m, \theta; K) \equiv c_2^{DI}(m, \theta; K) - c_1^{DI}(m, \theta; K). \tag{8}$$

Note that as the policymaker raises deposit insurance limit K, the incremental payoff $\Delta c(m, \theta; K)$ increases for any m and any θ. That is, deposit insurance enhances depositors' incentive to stay and hence deters bank runs. Figure 2 visually illustrates the incremental payoff function when $K < 1$ and $K \geq 1$.

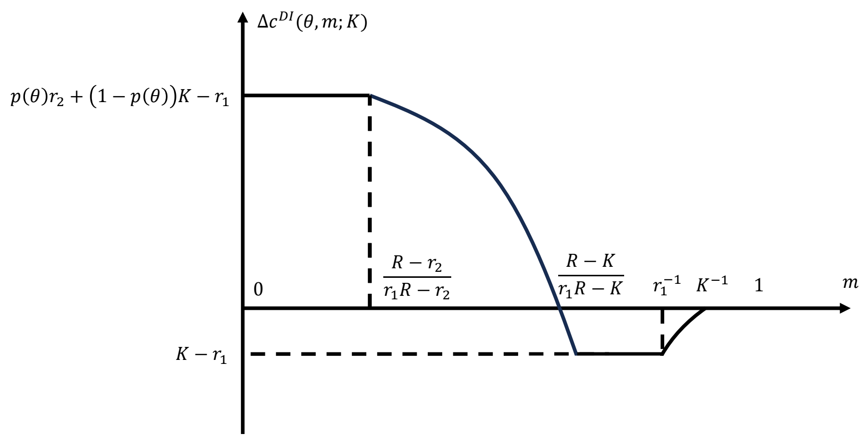
We analyze depositors' run incentives starting from the two extreme regions. If a depositor's belief about $p(\theta)$ is so low such that $\mathbb{E}[p(\theta)r_2 + (1-p(\theta))K | s_i] \leq r_1$, running is a dominant strategy. This requires $\mathbb{E}[p(\theta)|s_i] \leq \frac{r_1 - K}{r_2 - K}$. Since $p(\theta)$ increases in θ, and $\lim_{\theta \to -\infty} p(\theta) = 0$, there exists a threshold $\underline{s}$ such that $\mathbb{E}[p(\theta)|s_i] \leq \frac{r_1 - K}{r_2 - K}$ for $s_i \leq \underline{s}$. We call the region of $(\infty, \underline{s}]$ the *Lower Dominance Region*.[7]

On the other extreme, depositors' run incentives diminish with θ. We follow Goldstein and Pauzner (2005) and modify the investment technology in a way such that when $\theta > \bar{\theta}$, the bank's asset has a return of at least r_1 when liquidated at $t = 1$. This assumption can be justified by the existence of a liquid secondary market for long-term investment projects in extremely good economic conditions. Under this assumption, for $\theta > \bar{\theta}$, a late withdrawer is guaranteed

[7]When $K = r_1$, the Lower Dominance Region doesn't exist. In this case, staying is a dominant strategy for depositors regardless of fundamental θ. As it will become clear later, it is never optimal for the policymaker to set $K = r_1$.



(a) $K < 1$



(b) $K \geq 1$

Figure 2: Incremental Payoff From Staying (Deposit Insurance Alone)

a payoff of r_2 if the bank's asset pays off R. Since $\lim_{\theta \to +\infty} p(\theta) = 1$, there exists a threshold $\bar{s}$, such that for $s_i \in [\bar{s}, +\infty)$, staying is a dominant strategy for patient depositors. We call the region $[\bar{s}, +\infty)$ the *Upper Dominance Region*. We also assume $\bar{\theta}$ is arbitrarily large so that the quantitative implication of this additional assumption on depositors' equilibrium run decision and social welfare is negligible.

In the intermediate region of θ, as Figure 2 shows, the payoff structure features one-sided strategic complementarities. When the measure of withdrawal $m \leq 1/r_1$, the incremental payoff function is decreasing in m. That is, if more depositors withdraw, the bank is forced to liquidate more assets prematurely, boosting the incentive to withdraw for all patient depositors. However, when $m > 1/r_1$, the opposite is true. The bank has already liquidated all assets. Therefore, if more depositors withdraw, the payoff from withdrawing gets diluted, lowering the incentive to withdraw. This is a typical feature of bank-run models as illustrated in Goldstein and Pauzner (2005).

In a monotone perfect Bayesian equilibrium, depositors' withdrawal strategies are decreasing in their private signals. Therefore, in equilibrium, there is a threshold such that a depositor will stay with the bank if and only if his private signal about the bank fundamentals lands above the threshold. Given such a threshold $\hat{s}$, in a state with aggregate liquidity need λ and bank fundamentals θ, the measure of the withdrawals at $t = 1$ can be characterized as

$$m(\lambda, \theta; \hat{s}) = \lambda + (1 - \lambda)\Phi\left(\sqrt{\beta}\,(\hat{s} - \theta)\right). \tag{9}$$

Hence, the expected incremental payoff from staying for depositor i who observes a private signal s_i is given by

$$V^{DI}\left(s_i; K, \hat{s}\right) = \int_{-\infty}^{\infty}\int_0^1 \Delta c^{DI}\left(m(\lambda, \theta; \hat{s}), \theta; K\right) f(\theta|s_i) d\lambda d\theta. \tag{10}$$

A threshold depositor with a private signal $\hat{s}$ is indifferent between withdrawing at $t = 1$ and staying until $t = 2$. As a result, his expected incremental payoff is zero, i.e. $V^{DI}\left(\hat{s}; K, \hat{s}\right) = 0$. Despite the lack of global strategic complementarities, the single crossing property of the payoff difference function implies a unique optimal threshold strategy, as summarized in Lemma 1.

Lemma 1 *When the policymaker commits not to inject liquidity, there exists $\hat{\beta} > 0$ such that for any $\beta > \hat{\beta}$ and given any deposit insurance policy $K \in [0, r_1]$, there is a unique threshold strategy equilibrium. Specifically, for any $i \in [0, 1]$,*

$$d_i^{DI} = \begin{cases} 1, & \text{if } s_i \leq \hat{s}^{DI}; \\ 0, & \text{if } s_i > \hat{s}^{DI}, \end{cases} \tag{11}$$

where $\hat{s}^{DI} = -\infty$ if $K = r_1$, and $\hat{s}^{DI}$ is the unique solution to equation $V^{DI}\left(\hat{s}; K, \hat{s}\right) = 0$ if $K \in [0, r_1)$. Moreover, depositors' run threshold $\hat{s}$ strictly decreases in K.

3.2 Optimal Deposit Insurance Policy

Given depositors' optimal run strategy, we can then express the policymaker's payoff and her optimization problem. Specifically, her payoff at $t = 1$ in a state with bank fundamentals θ and aggregate liquidity need λ is given by

$$u_g^{DI}(\theta, \lambda; \hat{s}) = \max\left\{1 - r_1 m(\theta, \lambda; \hat{s}), 0\right\}\left(P(\theta)R - 1\right), \tag{12}$$

where $m(\theta, \lambda; \hat{s})$ is the measure of early withdrawal defined in Equation (9). Recall that transfers between the policymaker and any other agent in the economy do not impact the policymaker's payoff, i.e. the welfare of the economy. Therefore, the choice of deposit insurance policy matters only through its impact on $\hat{s}$, depositors' run threshold.

Lemma 1 implies that the policymaker can effectively achieve a target run threshold by choosing the corresponding deposit insurance policy. The result is summarized formally in Corollary 1.

Corollary 1 *For any $\hat{s} \in (-\infty, s_0^*]$, there exists a unique deposit insurance policy K such that depositors' optimal run threshold $\hat{s}^{DI} = \hat{s}$, where s_0^* is such that $V^{DI}(\hat{s}_0; 0, \hat{s}_0) = 0$, i.e., depositors' laissez-faire run threshold without any intervention policy.*

Therefore, the policymaker effectively chooses an optimal run threshold to maximize the welfare of the economy. That is,

$$\max_{\hat{s} \in (-\infty, s_0^*]} \mathbb{E}\left[u_g^{DI}(\theta, \lambda; \hat{s})\right]. \tag{13}$$

Suppose that the policymaker decides to lower depositors' run threshold $\hat{s}$ with a higher deposit insurance limit K. Equation (9) implies that the measure of withdrawal m decreases in all states of the economy. This is efficient when $P(\theta)R > 1$ as it preserves more positve-NPV investment by the bank. However, when $P(\theta)R < 1$, this is inefficient, because it reduces liquidation of negative-NPV investment by the bank. The optimal policy trades off these two forces such that the marginal effect is zero, which we summarize in 2.

Lemma 2 *When the policymaker commits not to inject liquidity, her optimal deposit insurance policy K_{DI}^* is such that depositors' run threshold s_{DI}^* solves $\frac{d}{d\hat{s}}\mathbb{E}\left[u_g^{DI}(\theta, \lambda; \hat{s})\right] = 0$. In the limit of vanishing information friction, $\lim_{\beta \to \infty} s_{DI}^* = p^{-1}\left(R^{-1}\right)$.*

Without ex-post liquidity injection, the policymaker optimally implements a target threshold such that the marginal long-term investment of the bank has zero NPV. The standard intuition applies. The optimal deposit insurance policy minimizes inefficient panic runs while allowing fundamental runs to force the bank to liquidate investment with negative NPV. However, deposit insurance alone does not allow the policymaker to provide ex-post liquidity support, which leads to inefficiencies especially when the aggregate liquidity need λ is large.

4 Policy Portfolio

In this section, we analyze the dynamic interactions between deposit insurance and liquidity injection and derive the equilibrium policy portfolio. We show that in equilibrium, the policymaker commits to a zero deposit insurance. Interestingly, compared with the case in which the policymaker commits not to inject liquidity, when the policymaker can freely design policy portfolios, she strategically allows more panic bank runs to effectively direct liquidity injections.

4.1 Policymaker's Learning and Liquidity Injection

We first explore the policymaker's inference about the bank's fundamentals based on the total demand for early withdrawal. Since the depositors' run strategies are decreasing in their private signals as required in Definition 1, they will employ a symmetric cutoff strategy with the threshold $\hat{s}$ in equilibrium: any depositor i withdraws if and only if $s_i \leq \hat{s}$. As characterized in Equation (9), given depositors' run threshold $\hat{s}$, the demand for early withdrawal m depends on the realization of two shocks: the bank's fundamentals and the aggregate liquidity need. Hence, a large demand for withdrawal m can occur due to a large aggregate liquidity need λ, reflecting withdrawals by impatient depositors, or weak bank fundamentals θ, prompting patient depositors to run on the bank. Therefore, observing only the total withdrawal m, the policymaker cannot perfectly distinguish the cause.

The policymaker makes inference about bank fundamentals θ based on the measure of withdrawals at $t = 1$, with a conditional cumulative distribution function (on m and $\hat{s}$) of the bank fundamentals given by

$$F_\theta(\theta|m;\hat{s}) = \begin{cases} \dfrac{\int_{\underline{\theta}(m;\hat{s})}^{\theta} \frac{\phi(x)}{1-n(x;\hat{s})}\mathrm{d}x}{\int_{\underline{\theta}(m;\hat{s})}^{\infty} \frac{\phi(x)}{1-n(x;\hat{s})}\mathrm{d}x} & \text{if } \theta \geq \underline{\theta}(m;\hat{s}); \\ 0 & \text{otherwise,} \end{cases} \tag{14}$$

where

$$n(\theta;\hat{s}) = \Phi\left(\sqrt{\beta}\,(\hat{s}-\theta)\right) \tag{15}$$

represents the fraction of patient depositors requesting to withdraw at $t = 1$, and $\underline{\theta}(m;\hat{s}) = \hat{s} - \frac{1}{\sqrt{\beta}}\Phi^{-1}(m)$ is the lower bound for θ such that m is feasible. For $\theta < \underline{\theta}(m;\hat{s})$, the fraction of patient depositors demanding early withdrawal is greater than m, leading to a total withdrawal greater than m. The derivation can be found in the proof of Proposition 1.

Intuitively, a larger withdrawal m indicates weaker bank fundamentals. Importantly, the depositors' run strategies significantly influence the policymaker's inference. It follows from Equation (15) that for any given bank fundamentals θ, an increase in $\hat{s}$ leads to more withdrawals by patient investors. Consequently, with a larger $\hat{s}$, observing the same total withdrawal m, the policymaker believes that the average bank fundamentals are stronger. The policymaker's inferences are formally summarized in Proposition 1.

Proposition 1 *For any $m_1 < m_2$, $F_\theta(\theta|m_1;\hat{s})$ has first-order stochastic dominance over $F_\theta(\theta|m_2;\hat{s})$. For any $\hat{s}_1 > \hat{s}_2$, $F_\theta(\theta|m;\hat{s}_1)$ has first-order stochastic dominance over $F(\theta|m;\hat{s}_2)$.*

Given her posterior belief about the bank fundamentals in Equation (14), the policymaker makes her liquidity injection decision. In particular, after observing the measure of withdrawal m, the policymaker chooses ℓ to maximize her expected payoff

$$\max_{\ell\in[0,mr_1]} u_g^{LI}(l;m,\hat{s}) = \max\{1+\ell-r_1m,0\}\left(\mathbb{E}(P(\theta)|m;\hat{s})R-1\right), \tag{16}$$

where

$$\mathbb{E}\left(P(\theta)|m;\hat{s}\right) = \int_{-\infty}^{+\infty} P(\theta)\mathrm{d}F(\theta|m;\hat{s}). \tag{17}$$

Since the policymaker is risk-neutral, it follows that the policymaker optimally injects the maximum amount of liquidity, if the bank's long-term investment has a positive NPV in expectation. By contrast, if the bank's investment has a negative NPV in expectation, the policymaker injects zero liquidity. Such a result is formally summarized in Lemma 3.

Lemma 3 *Given the depositors' withdrawal strategies $\hat{s}$, the optimal liquidity injection conditional on the measure of withdrawal m is*

$$\mathcal{L}^*(m,\hat{s}) = \begin{cases} 0, & \text{if } \mathbb{E}\left(P(\theta)|m;\hat{s}\right) \leq \frac{1}{R}; \\ mr_1, & \text{if } \mathbb{E}\left(P(\theta)|m;\hat{s}\right) > \frac{1}{R}. \end{cases} \tag{18}$$

Proposition 1 implies that upon observing a larger measure of withdrawal m, the policymaker forms a more pessimistic belief about bank fundamentals θ. Therefore, the zero NPV threshold in Equation (18) can be reformulated into a condition on the measure of withdrawal m. Specifically, given any $\hat{s}\in\mathbb{R}$, we define the liquidity injection cutoff $\hat{m}^{BR}\in(0,1)$ such that

$$\mathbb{E}\left(P(\theta)|\hat{m}^{BR};\hat{s}\right) = \int_{-\infty}^{+\infty} P(\theta)\mathrm{d}F(\theta|\hat{m}^{BR};\hat{s}) = \frac{1}{R}. \tag{19}$$

Corollary 2 then states the optimal liquidity injection policy contingent on the observed measure of withdrawal m.

Corollary 2 *For any given run threshold $\hat{s}$, there is a unique $\hat{m}^{BR}$ that solves Equation (19) and is strictly increasing in $\hat{s}$. Then, the policymaker's equilibrium liquidity injection strategy is*

$$\mathcal{L}^*(m, \hat{s}) = \begin{cases} 0, & \text{if } m \geq \hat{m}^{BR}; \\ mr_1, & \text{if } m < \hat{m}^{BR}. \end{cases} \tag{20}$$

Corollary 2 implies that the policymaker tends to provide support to banks experiencing a small amount of early withdrawals, when she does not know the bank fundamentals. This makes sense since a small amount of early withdrawals suggests that the bank fundamentals are strong. More remarkably, the optimal liquidity injection threshold $\hat{m}^{BR}$ strictly increases in $\hat{s}$. This is because if depositors adopt more aggressive run strategies, runs are more likely to be the result of panic rather than weak bank fundamentals. Hence, the policymaker will have a more optimistic inference about θ, increasing her incentive to inject liquidity.

Corollary 2 also suggests a useful tool, the *iso-m curve*, which visualizes our analyses and helps clarify the underlying intuitions. In particular, a state of the economy is characterized by two state variables – aggregate liquidity need λ and bank fundamentals θ. On the space of λ and θ, given any $\hat{s} \in \mathbb{R}$, an iso-m curve charts all the states with the same total measure of withdrawal m. That is, an iso-m curve depicts all the combinations of λ and θ that satisfy Equation (9). For example, Figure 3 plots the iso-$\hat{m}^{BR}$ curve, which consists of all states with a total measure of withdrawal equal to the liquidity injection cutoff $\hat{m}^{BR}$. However, the underlying withdrawal motives are different along the curve. In particular, any iso-m curve is upward-sloping. When bank fundamentals are strong, most withdrawals are made by impatient depsitors due to liquidity needs, reflected by a combination of large λ and large θ. In contrast, when bank fundamentals are weak, most withdrawals are made by patient depositors due to fundamental-related panic, reflected by a combination of a small λ and a small θ.

In states that are to the right and below the iso-$\hat{m}^{BR}$ curve, given an aggregate liquidity shock λ, the bank fundamentals θ are so good that less patient depositors run, leading to a total measure of withdrawal $m < \hat{m}^{BR}$. Corollary 2 then implies that in these states, the policymaker optimally injects the maximum amount of liquidity. While, to the left and above the curve, the total measure of withdrawal $m > \hat{m}^{BR}$, so the policymaker optimally injects zero liquidity.

4.2 Panic Bank Run

Moving backward, we then analyze depositors' withdrawal strategy, anticipating the optimal liquidity injection by the policymaker characterized in Section 4.1. As shown in Corollary 2,

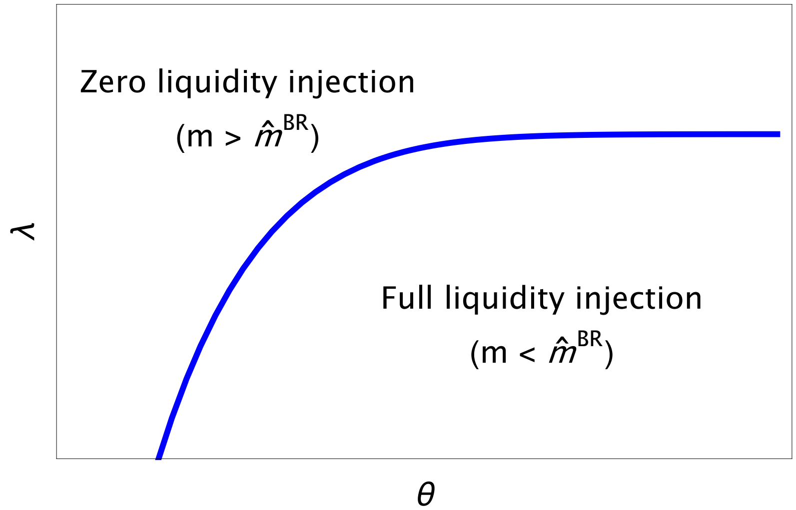


Figure 3: Iso-$\hat{m}^{BR}$ Curve and Optimal Liquidity Injection Policy

the policymaker optimally follows a threshold liquidity injection strategy; that is, inject full liquidity when the mass of withdrawal is below a threshold $\hat{m}$ and inject zero liquidity otherwise. Therefore, given a deposit insurance policy K and a liquidity injection threshold $\hat{m}$, we can express any depositor i's payoff from withdrawing at $t = 1$ as

$$c_1(m, \theta; K, \hat{m}) = \begin{cases} K, & \text{if } m \geq \max\left\{K^{-1}, \hat{m}\right\}; \\ \frac{1}{m}, & \text{if } m \geq \hat{m} \text{ and } K^{-1} > m \geq r_1^{-1}; \\ r_1, & \text{otherwise.} \end{cases} \tag{21}$$

Similarly, depositor i's payoff from staying with the bank until $t = 2$ can be expressed as

$$c_2(m, \theta; K, \hat{m}) = \begin{cases} K, & \text{if } m \geq \max\left\{\frac{R-K}{r_1 R - K}, \hat{m}\right\}; \\ p(\theta)\frac{(1-mr_1)R}{1-m} + (1-p(\theta))K, & \text{if } m \geq \hat{m} \text{ and } \frac{R-K}{r_1 R - K} > m \geq \frac{R-r_2}{r_1 R - r_2}; \\ p(\theta) r_2 + (1-p(\theta))K, & \text{otherwise.} \end{cases} \tag{22}$$

To visualize the impact of liquidity injection and deposit insurance on depositors' run incentives, we plot the incremental payoff from staying defined below

$$\Delta c\,(m, \theta; K, \hat{m}) = c_2(m, \theta; K, \hat{m}) - c_1(m, \theta; K, \hat{m}) \tag{23}$$

in Figure 4. Compare this figure with Figure 2 which corresponds to the benchmark with deposit insurance alone. The only difference is that if the total measure of withdrawal $m < \hat{m}$, then the staying depositors are protected from the losses caused by premature liquidation of

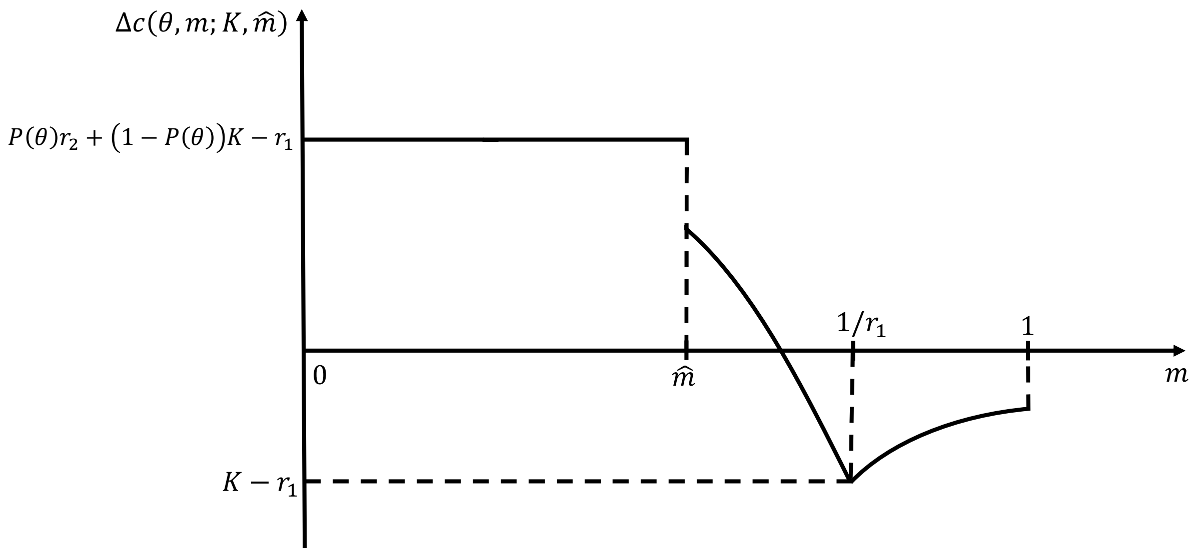


Figure 4: Incremental Payoff from Staying (Policy Portfolio)

the bank's long-term investment. Therefore, as long as $\hat{m} > \frac{R-r_2}{r_1 R - r_2}$, patient depositors have less incentive to run in anticipation of ex-post liquidity injection. Deposit insurance also deters runs by patient investors yet functions through a different channel – it is an ex-ante proactive guarantee as opposed to an ex-post reactive measure.

Analagous to the benchmark with deposit insurance alone, patient depositors optimally follow a threshold strategy, and the signal threshold is characterized by the indifference condition of a threshold depositor. Specifically, the expected incremental payoff from staying for depositor i with a private signal s_i can be expressed as

$$V(s_i; K, \hat{s}) = \int_0^1 \int_{-\infty}^{\infty} \Delta c \left(m(\lambda, \theta; \hat{s}), \theta; K, \hat{m}^{BR} \right) f(\theta | s_i) d\theta d\lambda \tag{24}$$

where $m(\lambda, \theta; \hat{s})$ is the measure of withdrawals at $t = 1$ given by Equation (9), and $f(\theta|s_i)$ is the posterior p.d.f. of θ given by Equation (33). Since the incremental payoff function satisfies the single-crossing condition, there exists a unique run threshold.

Lemma 4 *When the policymaker cannot commit not to injecting liquidity, given any deposit insurance policy $K \in [0, r_1]$, depositors follow a threshold strategy in equilibrium. Specifically, for any $i \in [0, 1]$,*

$$d_i^* = \begin{cases} 1, & \text{if } s_i \leq \hat{s}; \\ 0, & \text{if } s_i > \hat{s}, \end{cases} \tag{25}$$

where $\hat{s} = -\infty$ if $K = r_1$ and $\hat{s}$ is the unique solution to equation $V(\hat{s}; K, \hat{s}) = 0$ if $K \in [0, r_1)$. Moreover, depositors' run threshold $\hat{s}$ strictly decreases in K.

Since depositors' optimal run threshold $\hat{s}$ strictly decreases in K, we have immediately Corollary 3 below.

Corollary 3 *For any $s^* \in (-\infty, \hat{s}_0]$, there exists a unique deposit insurance policy K such that the depositors' signal threshold $\hat{s} = s^*$, where the upper bound $\hat{s}_0$ is such that $V(\hat{s}_0; 0, \hat{s}_0) = 0$, i.e., depositors' signal threshold without deposit insurance.*

4.3 Optimal Policy Portfolio

We are now at a position to analyze the policymaker's deposit insurance policy K, accounting for its impact on depositors' withdrawal strategy characterized by Lemma 4.

Given any run threshold $\hat{s} \in \mathbb{R}$, we can express the policymaker's payoff at $t = 1$ after the realization of banks' fundamentals θ and aggregate liquidity need λ. In particular, if the liquidity injection threshold $\hat{m} < r_1^{-1}$,

$$u_g(\theta, \lambda; \hat{s}, \hat{m}) = \begin{cases} 0 & \text{if } m(\theta, \lambda; \hat{s}) \geq r_1^{-1}; \\ (1 - r_1 m(\theta, \lambda; \hat{s}))\,(p(\theta)R - 1) & \text{if } r_1^{-1} > m(\theta, \lambda; \hat{s}) \geq \hat{m}; \\ p(\theta)R - 1 & \text{otherwise.} \end{cases} \tag{26}$$

If the liquidity injection threshold $\hat{m} \geq r_1^{-1}$,

$$u_g(\theta, \lambda; \hat{s}, \hat{m}) = \begin{cases} 0 & \text{if } m(\theta, \lambda; \hat{s}) \geq \hat{m}; \\ p(\theta)R - 1 & \text{otherwise.} \end{cases} \tag{27}$$

Recall that the deposit insurance policy influences welfare solely through its impact on depositors' run threshold. Consequently, when the policymaker decides on the deposit insurance policy, she is effectively determining the optimal run threshold for depositors. As stated in Corollary 3, any threshold $\hat{s} \in (-\infty, \hat{s}_0)$ can be achieved with a deposit insurance policy $K \in [0, r_1]$. Therefore, the policymaker is effectively determining the optimal run threshold to maximize social welfare. We can formulate her optimization problem as

$$\max_{\hat{s} \in (-\infty, \hat{s}_0)} \mathbb{E}\left[u_g(\theta, \lambda; \hat{s}, \hat{m}^{BR})\right] \tag{28}$$

where $\hat{m}^{BR}$ is her optimal liquidity injection threshold defined by Equation (19).

It turns out that when both policy tools are allowed, the policymaker finds it optimal to use only ex-post liquidity injection and does not provide any ex-ante deposit insurance.

Proposition 2 *When the policy maker cannot commit not to injecting liquidity, the optimal deposit insurance policy is* $K^* = 0$. *Depositors follow a threshold strategy given by Equation* (25) *with a threshold* $s^* = \hat{s}_0 \in (s^*_{DI}, s^*_0)$. *The optimal policy portfolio therefore consists only of a threshold liquidity injection policy given by Equation* (20) *with a threshold* m^* *such that* $\mathbb{E}(P(\theta)|m^*; s^*) = 1/R$.

Detailed analyses are relegated to the proof in the appendix. Here, we outline the intuition. Consider the welfare implication of increasing $\hat{s}$ marginally; that is, the policymaker reduces the coverage of deposit insurance to allow more bank runs. The effect is positive on both intensive and extensive margins.

Intensive Margin Figure 5 illustrates the effect on the intensive margin. A marginal increase in $\hat{s}$ leads to an increase in early withdrawal m across all states of the world. In the region below Iso-$\hat{m}$ curve (point A for example), this is inconsequential because the policymaker carries out full liquidity injection to offset the increase in withdrawal demand. Ultimately, the bank does not liquidate any long-term investment in this region. Similarly, in the region above Iso-r_1^{-1} curve (point B for example), the bank has liquidated all long-term investment, which is still insufficient to fulfill the withdrawal demand. Hence, an increase in m has no impact on welfare in this region.

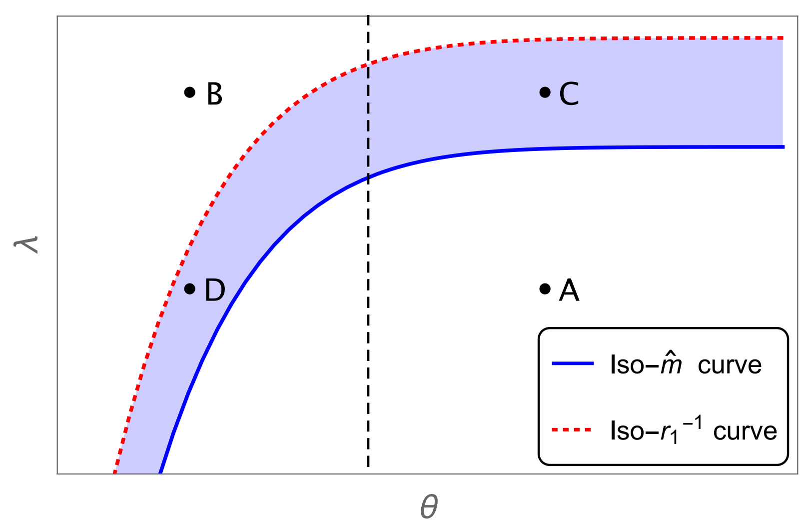


Figure 5: Effect of Increasing $\hat{s}$ on the Intensive Margin

An increase in withdrawal m would force the bank to liquidate more of its long-term investment and have a real impact in the shaded region between the two iso-m curves. The dashed vertical line represents the zero NPV fundamental cutoff, i.e. $\theta = P^{-1}(R^{-1})$. In bad states with low bank fundamentals θ (point D for example), this improves efficiency because the bank's investment has a negative NPV. However, it is detrimental when θ is high (point C for example) since the bank investment has a positive NPV. As we explain next, the effect is more pronounced in bad states than in good states, resulting in a net gain in welfare.

Why is the effect less pronounced in good states with high bank fundamentals θ? In a state (λ, θ), the total measure of withdrawal is given by Equation (9), which we repeat below.

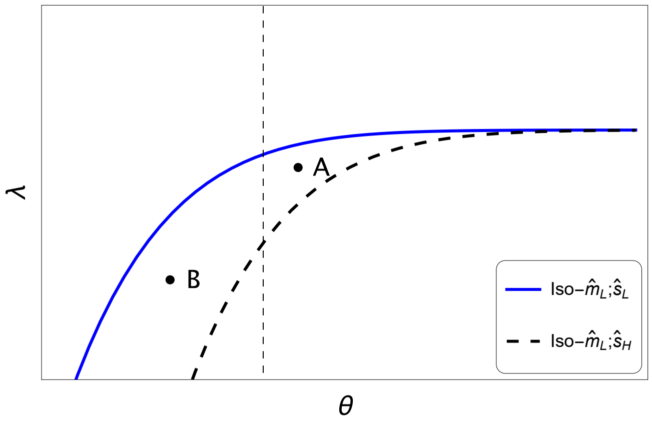
$$m = \lambda + (1 - \lambda)n(\theta; \hat{s})$$

where $n(\theta; \hat{s}) = \Phi\left(\sqrt{\beta}(\hat{s} - \theta)\right)$ is the fraction of patient depositors demanding early withdrawal. Recall that iso-m curves are upward-sloping representing different compositions of withdrawal motives. When bank fundamentals are strong, most of the withdrawals are made by impatient depositors due to liquidity needs, reflected by a combination of large λ and large θ. An increase in $\hat{s}$ leads to more withdrawals by patient depositors, i.e., $n(\theta; \hat{s})$ increases. However, in these states, the measure of patient depositors $1 - \lambda$ is small anyway. Thus, more withdrawals by these patient depositors have a minimal impact.

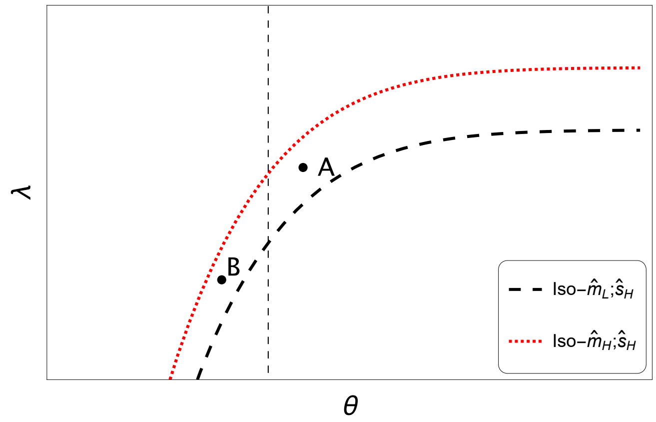
The positive effect on the intensive margin is at play as long as $\hat{m} < r_1^{-1}$. If $\hat{m} \geq r_1^{-1}$, the region between the two iso-m curves in Figure 5 no longer exists, and hence, there is no welfare impact on the intensive margin.

Extensive Margin Figure 6 illustrates the effect of increasing run threshold from $\hat{s}_L$ to $\hat{s}_H$ in two steps.

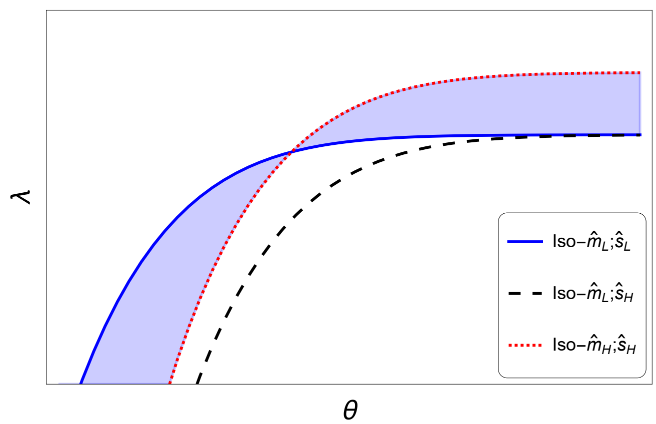
First, suppose the policymaker sticks to the same liquidity injection threshold $\hat{m}_L$. That is, she injects full liquidity if $m < \hat{m}_L$ and zero liquidity otherwise. The blue curve in Figure 6a portrays the initial liquidity injection boundary when patient depositors follow a conservative run strategy with a threshold $\hat{s}_L$. As depositors adopt a more aggressive run strategy with a threshold $\hat{s}_H$, the measure of withdrawal increases in all states of the economy. Therefore, some states that originally received liquidity injection now surpass the threshold $\hat{m}_L$ and consequently lose liquidity support. The dashed black curve depicts the new liquidity injection boundary, and all states that fall between the two curves lose their liquidity support. This is welfare-enhancing when bank fundamentals are weak (point B for example) because the bank is forced to liquidate its negative NPV investment. However, when bank investment has positive NPV (point A for example), losing liquidity injection is welfare-destroying. When bank fundamentals are strong (point A), the measure of patient depositors $1 - \lambda$ is small to begin with. Hence, the increase of withdrawals by patient depositors has a minimal impact on the



(a) Change $\hat{s}$ Holding $\hat{m}$ Fixed



(b) Change $\hat{m}$



(c) Aggregate Effect

Figure 6: Effect of Increasing $\hat{s}$ on the Extensive Margin

total withdrawal m. As a result, there are fewer good states with strong bank fundamentals that lose liquidity injection than bad states, generating a net gain in welfare.

Second, we take into account the change in the liquidity injection threshold. As we just discussed, an increase in run threshold helps the policymaker to tease out more bad states with poor bank fundamentals. Therefore, the policymaker makes more optimistic inference about bank fundamentals and becomes more willing to inject liquidity. Consistent with Corollary 2, the optimal liquidity injection threshold would increase from $\hat{m}_L$ to $\hat{m}_H$, as illustrated by Figure 6b. The dotted red curve depicts the new liquidity injection boundary. Relative to the black dashed curve, it shifts up and to the left, and the bank now receives liquidity injection in the region between the two iso-m curves. Again, the welfare implication depends on the bank fundamentals. When the bank's long-term investment has positive NPV (point A for example), it is welfare-enhancing to inject liquidity. It is welfare-destroying to injection liquidity when the bank's investment has negative NPV (point B for example). However, according to the envelop theorem, the welfare gain and loss net out. Mathematically, this effect is captured by $\frac{\partial \mathbb{E}(u_g)}{\partial \hat{m}}|_{\hat{m}^{BR}} \frac{\partial \hat{m}^{BR}}{\partial \hat{s}}$, where $\frac{\partial \mathbb{E}(u_g)}{\partial \hat{m}}|_{\hat{m}^{BR}} = 0$ due to the ex-post optimality of liquidity injection policy. Hence, the increase in $\hat{m}$ has zero marginal effect on the welfare.

Figure 6c displays the aggregate effect on the extensive margin. Comparing the solid blue curve with the red dotted curve, we see vividly that liquidity injection becomes more efficient: it rescues more states with high bank fundamentals and hence positive-NPV investment, and it gives up more states with low bank fundamentals and hence negative-NPV investment. This positive effect on the extensive margin is at play for any $\hat{m}$.

Since increasing $\hat{s}$, thus allowing more bank runs, has a positive effect on both the intensive and extensive margins, we arrive at a powerful result that increasing run threshold of depositors is always beneficial in the presence of ex post liquidity injection. Therefore, the optimal deposit insurance should be zero to maximize bank runs. In this sense, ex ante deposit insurance is overshadowed by ex post liquidity injection.

Information Precision If depositors receive more precise information, their runs contain more information, which helps the policymaker to further improve the efficiency of ex post liquidity injection. We illustrate this effect in Figure 7, which compares the optimal liquidity injection boundaries when depositors receive vague signals (small β) with that when they receive precise signals (large β).

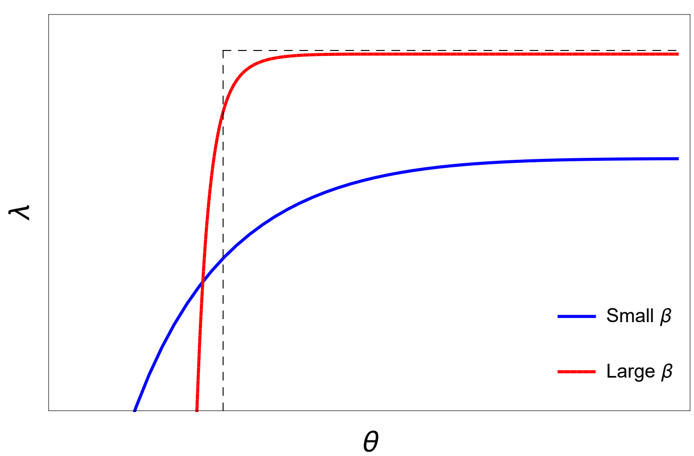


Figure 7: Liquidity Injection Boundaries Varying Information Precision β

As depositors' information gets more precise, the liquidity injection boundary gets closer to the perfect information case represented by the dashed black curve. We analyze formally this case in Appendix A. If the policymaker perfectly observes bank fundamentals θ, her optimal liquidity injection policy is to inject full liquidity when the bank's investment has positive NPV and inject zero liquidity when the bank's investment has negative NPV.

In the limit of negligible information friction, the liquidity injection boundary converges to the perfect information case. Recall that in the deposit-insurance-alone benchmark, the policymaker implements a run threshold at which the bank's long-term investment has zero NPV. In other words, deposit insurance can eliminate all inefficient panic runs when the bank's investment has positive NPV. In comparison, the optimal policy portfolio implements a higher run threshold for depositors. Therefore, inefficient panic-based run can occur. However, these panic runs are fully rescued ex post with liquidity injection, and therefore, first best is achieved. We summarize the limit case in Lemma 5.

Lemma 5 *In the limit of negligible information friction ($\beta \to \infty$), the equilibrium liquidity injection threshold* $\lim_{\beta\to\infty} m^* = 1$, *and depositors' run threshold* $\lim_{\beta\to\infty} s^* > \lim_{\beta\to\infty} s^*_{DI} = P^{-1}(1/R)$. *The optimal policy portfolio achieves the first best.*

The comparison with the deposit-insurance-alone benchmark yields interesting insights on policy design. First, deposit insurance and liquidity injection have different emphases. Deposit insurance is more effective in enhancing *financial stability* thanks to its ex ante commitment of guaranteeing a minimal payment to the depositors. Liquidity injection is more effective in improving *resource allocation efficiency* given its ex post flexibility and contingency on bank fundamentals (although imperfectly). Second, enhancing financial stability and resource allocation efficiency are two distinct goals. Although they are related, they are not always aligned. Liquidity injection enables the policymaker to mitigate real damages caused by panic runs, which reduces the need to stablize the bank ex ante with deposit insurance. We find that the optimal policy portfolio sacrifices financial stability by allowing more panic runs such that higher real efficiency can be achieved.

5 Conclusion

Banking crises persist as significant threats to our economy, necessitating ongoing policy interventions to bolster financial stability and enhance resource allocation efficiency. Among these interventions, deposit insurance and liquidity injection are commonly employed. This paper investigates the dynamic interaction between deposit insurance and liquidity injection.

Individually, both deposit insurance and liquidity injection contribute to financial stability and resource allocation efficiency. Deposit insurance reduces depositor incentives to withdraw, thereby mitigating panic bank runs and allowing banks to maintain liquidity in long-term investments, potentially improving resource allocation efficiency. Conversely, liquidity injection responds to withdrawal sizes, potentially enhancing resource allocation by reducing panic bank runs through depositor anticipation of potential liquidity support.

Integrated into a policy portfolio, however, the two policies do not exhibit positive synergy. In equilibrium, the policymaker will commit to a zero deposit insurance. First, for banks with strong fundamentals, the policymaker is likely to inject full liquidity to keep their long-term investments intact, hence the panic runs are not a severe concern in this case. For banks with weak fundamentals, the more panic runs the higher the resource allocation efficiency. Hence, when the policymaker commits to a higher the deposit insurance limit, there will be less panic runs at banks with weak fundamentals, leading to a less efficient resource allocation. Second, a lower deposit insurance limit leads to more panic runs, so for any liquidity shock, the bank needs to have a better fundamentals to get liquidity support from the government. Such an effect is more salient for banks with weak fundamentals. Therefore, a lower deposit insurance limit has a positive effect on resource allocation efficiency through this channel too.

This paper contributes both theoretically and practically. From a theoretical perspective, we develop novel approaches to solve a global-game-based model incorporating two policy measures. From an applied perspective, we demonstrate that a higher deposit insurance limit may result in reduced liquidity injection, a hypothesis amenable to empirical testing. Importantly, our findings suggest that deposit insurance should be carefully calibrated, particularly when the policymaker cannot commit to refraining from ex-post liquidity injection.

As a concluding remark, we streamline the model to highlight the central message that liquidity injection overshadows deposit insurance, underscoring the importance of a holistic approach that considers the interactions between different policy tools. In our model, this force drives the optimal level of deposit insurance down to zero. In reality, however, other factors may render deposit insurance a valuable policy. For example, large-scale liquidity injections can be politically contentious and may impose significant costs on taxpayers. Furthermore, the lack of commitment can expose liquidity injections to moral hazard problems, potentially making them more expensive to implement than deposit insurance.

A Perfectly Informed Policymaker

This appendix establishes a benchmark where the policymaker is perfectly informed about the bank fundamentals when making liquidity injection at $t = 1$. In such a case, the policymaker does not need to learn about the bank fundamentals from the measure of withdrawals.

We solve the benchmark model by backward induction, starting with the policymaker's liquidity injection. Since the policymaker is perfectly informed about the bank fundamentals θ, it follows from Equation (5) that when $P(\theta)R > 1$, the policymaker optimally inject the maximum amount of liquidity; that is, $\ell = mr_1$. By contrast, if $P(\theta)R \leq 1$, the policymaker will not inject any liquidity, so $\ell = 0$. Such a result is formally summarized in Lemma 6.

Lemma 6 *When the policymaker is perfectly informed about the bank fundamentals θ at $t = 1$, the optimal liquidity injection is*

$$\mathcal{L}^I(\theta) = \begin{cases} 0, & \text{if } \theta \leq \theta^I \equiv P^{-1}\left(\frac{1}{R}\right); \\ mr_1, & \text{if } \theta > \theta^I. \end{cases} \tag{29}$$

Lemma 6 shows that the policymaker's liquidity injection decision based on whether the bank investment has a positive NPV. When $P(\theta)R > 1$, long-term investment is productive and hence bank runs are inefficient. In this case, the policymaker injects maximum liquidity to cover all inefficient bank runs. However, when $P(\theta)R \leq 1$, long-term investment is counterproductive and hence bank runs are efficient. Therefore, the policymaker will not inject any liquidity in this case.

Given the policymaker's liquidity injection strategy characterized in Lemma 6, any depositor i's payoff from requesting to withdraw at $t = 1$, conditional on any committed deposit insurance K and the measure of withdrawals at $t = 1$, is

$$c_1^I(m, \theta; K) = \begin{cases} K, & \text{if } \theta \leq P^{-1}\left(\frac{1}{R}\right) \text{ and } m \geq K^{-1}; \\ \frac{1}{m}, & \text{if } \theta \leq P^{-1}\left(\frac{1}{R}\right) \text{ and } K^{-1} > m \geq r_1^{-1}; \\ r_1, & \text{otherwise.} \end{cases} \tag{30}$$

Similarly, if depositor i decides to stay with the bank until $t = 2$, his payoff is

$$c_2^I(m, \theta; K) = \begin{cases} K, & \text{if } \theta \leq P^{-1}\left(\frac{1}{R}\right) \text{ and } m \geq \frac{R-K}{r_1 R - K}, \\ P(\theta)\frac{(1-mr_1)R}{1-m} + (1-P(\theta))K, & \text{if } \theta \leq P^{-1}\left(\frac{1}{R}\right) \text{ and } \frac{R-K}{r_1 R - K} > m \geq \frac{R-r_2}{r_1 R - r_2}, \\ P(\theta) r_2 + (1-P(\theta))K, & \text{otherwise.} \end{cases} \tag{31}$$

In a monotone perfect Bayesian equilibrium, depositors' withdrawal strategies are decreasing in their private signals. Therefore, in equilibrium, there is a threshold such that a depositor will stay with the bank if and only if his private signal about the bank fundamentals lands above the threshold. Denote by s^I such a threshold, depositor i with a private signal s_i has the difference between his payoffs from staying with the bank until $t = 2$ and from withdrawing at $t = 1$ as

$$V^I(s_i; K, s^I) = \int_0^1 \int_{-\infty}^{\infty} \left[c_2^I \left(m(\lambda, \theta; s^I), \theta; K \right) - c_1^I \left(m(\lambda, \theta; s^I), \theta; K \right) \right] f(\theta | s_i) \mathrm{d}\theta \mathrm{d}\lambda \tag{32}$$

where

$$f(\theta | s_i) = \sqrt{1+\beta} \phi \left(\sqrt{1+\beta} \left(\theta - \frac{\beta s_i}{1+\beta} \right) \right) \tag{33}$$

is the posterior p.d.f. of θ for depositor i who observes the signal s_i, and $m(\lambda, \theta; \hat{s})$ is the measure of withdrawal given by Equation (9).

Lemma 7 then shows that unless $K = r_1$, there is a unique s^I such that $V^I(s^I; K, s^I) = 0$; that is, if the policymaker does not provide a full deposit insurance, a depositor will withdraw at $t = 1$ if and only if her private signal $s_i \leq s^I$.

Lemma 7 *When the policymaker is perfectly informed about the bank fundamentals θ at $t = 1$, depositors follow a threshold strategy in equilibrium. Specifically, for any $i \in [0, 1]$,*

$$d_i^I = \begin{cases} 1, & \textit{if } s_i \leq s^I; \\ 0, & \textit{if } s_i > s^I, \end{cases} \tag{34}$$

where $s^I = -\infty$ if $K = r_1$ and s^I is the unique solution to equation $V^I \left(s^I; K, s^I \right) = 0$ if $K \in [0, r_1)$. Moreover, depositors' run threshold s^I decreases in K.

Lemma 7 shows that although the deposit insurance payment, as a value transfer, does not directly affect the policymaker's payoff, it does affect the measure of runs at $t = 1$. In particular, a higher level of deposit insurance reduces early withdrawals in all states of the bank fundamentals. When $\theta > \theta^I$, early withdrawals are inefficient because the bank's investment has a positive NPV. However, such an inefficiency can be entirely eliminated through liquidity injection. Therefore, although deposit insurance reduces inefficient withdrawals, this benefit is overshadowed by liquidity injection. On the other hand, when $\theta \leq \theta^I$, early withdrawals are efficient. Deposit insurance is harmful in this case because it reduces efficient early withdrawals. As a result, a zero deposit insurance is dominant for the policy maker at $t = 0$.

Lemma 8 *When the policymaker is perfectly informed about the bank fundamentals θ at $t = 1$, the optimal deposit insurance is $K^I = 0$.*

Lemma 8 demonstrates that when the policymaker is perfectly informed before liquidity injection, ex-post liquidity injection is a superior policy tool than deposit insurance. Therefore, the optimal policy portfolio in this case is to employ ex-post liquidity injection only. Formally,

Proposition 3 *When the policymaker is perfectly informed about the bank fundamentals at $t = 1$, the optimal policy portfolio is $(K^I, \mathcal{L}^I)$, where $K^I = 0$ and $\mathcal{L}^I$ is characterized in Equation (29).*

B Proofs of Lemmas and Propositions

Proof of Lemma 1:

This lemma can be viewed as a special case of Proposition 8 in Allen, Carletti, Goldstein, and Leonelloe (2018) with $u(c) = c$. *Q.E.D.*

Proof of Lemma 3:

Given the policymaker's payoff (Equation (5)) is linear in ℓ, it is straightforward that she would choose maximum $\ell = mr_1$ if $\theta > \theta^I \equiv P^{-1}(R^{-1})$ and minimum $\ell = 0$ otherwise. *Q.E.D.*

Proof of Lemma 7:

For any $K \in [0, r_1)$, given equations (3) and (4), we can show that depositor i's incremental payoff from staying until $t = 2$ satisfies single crossing property. Denote the incremental payoff as $\Delta c^I(m, \theta; K) = c_2^I(m, \theta; K) - c_1^I(m, \theta; K)$. If $K \in [0, 1]$, for any $\theta \in \mathbb{R}$, there exists a $m^* \in [0, 1]$ such that $\Delta c^I(m, \theta; K) > 0$ if $m < m^*$ and $\Delta c^I(m, \theta; K) < 0$ if $m > m^*$. If $K \in (1, r_1)$, for any $\theta \in \mathbb{R}$, there exists a $m^* \in [0, 1]$ such that $\Delta c^I(m, \theta; K) > 0$ if $m < m^*$, $\Delta c^I(m, \theta; K) < 0$ if $m \in (m^*, K^{-1})$ and $\Delta c^I(m, \theta; K) = 0$ if $m \in [K^{-1}, 1]$. Specifically, the cutoff $m^* = 0$ if $P(\theta) \leq \frac{r_1 - K}{r_2 - K}$, and $m^* = \frac{(1-p(\theta))K - r_1 + p(\theta)R}{(1-p(\theta))K - r_1 + p(\theta)r_1 R}$ otherwise.

Moreover, the posterior distribution $f(\theta|s_i)$ satisfies monotone likelihood ratio property. That is, for any $s_H > s_L$, we have $\frac{f(\theta|s_H)}{f(\theta|s_L)}$ strictly increasing in θ. Therefore, there exists a unique solution to $V^I(s^I; K, s^I) = 0$.

If $K = r_1$, then for any $\theta \in \mathbb{R}$ and any $m \in [0, 1]$, $\Delta c^I(m, \theta; K) \geq 0$. Staying until $t = 2$ is a strictly dominating strategy, and therefore $s^I = -\infty$. *Q.E.D.*

Proof of Proposition 1:

Given depositors' run threshold $\hat{s}$, denote the posterior c.d.f. and p.d.f. of θ conditional on m as $F_\theta(\theta|m;\hat{s})$ and $f_\theta(\theta|m;\hat{s})$ correspondingly. Similarly, denote the conditional c.d.f. and p.d.f. of m given θ as $F_m(m|\theta;\hat{s})$ and $f_\theta(\theta|m;\hat{s})$ correspondingly.

The Bayes' theorem implies that

$$f_\theta(\theta|m;\hat{s}) \propto f_m(m|\theta;\hat{s})\phi(\theta). \tag{35}$$

Since λ and θ are independent, we have

$$F_m(m|\theta;\hat{s}) = Pr\left(\lambda < \frac{m-n(\theta;\hat{s})}{1-n(\theta;\hat{s})}\right) = \begin{cases} \frac{m-n(\theta;\hat{s})}{1-n(\theta;\hat{s})} & \text{if } \theta \geq \underline{\theta}(m;\hat{s}); \\ 0 & \text{otherwise,} \end{cases} \tag{36}$$

where $n(\theta;\hat{s}) = \Phi\left(\sqrt{\beta}\,(\hat{s}-\theta)\right)$ and $\underline{\theta}(m;\hat{s}) = \hat{s} - \sqrt{\beta}^{-1}\Phi^{-1}(m)$. Therefore, it follows that

$$f_\theta(\theta|m;\hat{s}) = \begin{cases} \frac{\frac{\phi(\theta)}{1-n(\theta;\hat{s})}}{\int_{\underline{\theta}(m;\hat{s})}^{\infty} \frac{\phi(\theta)}{1-n(\theta;\hat{s})}d\theta} & \text{if } \theta \geq \underline{\theta}(m;\hat{s}); \\ 0 & \text{otherwise,} \end{cases} \tag{37}$$

and that

$$F_\theta(\theta|m;\hat{s}) = \begin{cases} \frac{\int_{\underline{\theta}(m;\hat{s})}^{\theta} \frac{\phi(x)}{1-n(x;\hat{s})}dx}{\int_{\underline{\theta}(m;\hat{s})}^{\infty} \frac{\phi(x)}{1-n(x;\hat{s})}dx} & \text{if } \theta \geq \underline{\theta}(m;\hat{s}); \\ 0 & \text{otherwise.} \end{cases} \tag{38}$$

Take the derivative of $F(\theta|m;\hat{s})$ with respect to m.

$$\frac{d}{dm}F(\theta|m;\hat{s}) = -\frac{\int_{\theta}^{\infty} \frac{\phi(x)}{1-n(x;\hat{s})}dx \frac{\phi(\underline{\theta}(m;\hat{s}))}{1-m}}{\left(\int_{\underline{\theta}(m;\hat{s})}^{\infty} \frac{\phi(x)}{1-n(x;\hat{s})}dx\right)^2} \frac{d}{dm}\underline{\theta}(m;\hat{s}) > 0 \tag{39}$$

Therefore, for any $m_1 < m_2$, $F(\theta|m_1;\hat{s}) < F(\theta|m_2;\hat{s})$.

$$\begin{aligned} \frac{d}{d\hat{s}}F(\theta|m;\hat{s}) =& \frac{\int_{\underline{\theta}(m;\hat{s})}^{\theta} \int_{\theta}^{\infty} \left(h\left(\sqrt{\beta}\,(\hat{s}-x)\right) - h\left(\sqrt{\beta}\,(\hat{s}-y)\right)\right) \frac{\sqrt{\beta}\phi(x)}{1-n(x;\hat{s})} \frac{\phi(y)}{1-n(y;\hat{s})}dydx}{\left(\int_{\underline{\theta}(m;\hat{s})}^{\infty} \frac{\phi(x)}{1-n(x;\hat{s})}dx\right)^2} \\ &- \frac{\frac{\phi(\underline{\theta}(m;\hat{s}))}{1-m} \int_{\theta}^{\infty} \frac{\phi(x)}{1-n(x;\hat{s})}dx}{\left(\int_{\underline{\theta}(m;\hat{s})}^{\infty} \frac{\phi(x)}{1-n(x;\hat{s})}dx\right)^2} \end{aligned} \tag{40}$$

where $h(\cdot) = \frac{\phi(\cdot)}{1-\Phi(\cdot)}$ is the hazard rate function of standard normal distribution. Since $h(\cdot)$ is a strictly increasing function, we have $h\left(\sqrt{\beta}\,(\hat{s}-x)\right) - h\left(\sqrt{\beta}\,(\hat{s}-y)\right) < 0$ for any $x \in [\underline{\theta}(m;\hat{s}), \theta]$ and $y \in [\theta, \infty)$. It follows that $\frac{d}{d\hat{s}} F(\theta|m;\hat{s}) < 0$. Therefore, for any $s_1^* > s_2^*$, $F_\theta(\theta|m;s_1^*) < F_\theta(\theta|m;s_2^*)$. *Q.E.D.*

Proof of Lemma 3:

Since the policymaker's payoff (equation 5) is linear in $\mathbb{E}(P(\theta)|m;\hat{s})$, it is straightforward to show that the policymaker maximizes liquidity injection when the bank's investment has positive NPV, i.e. $\mathbb{E}(P(\theta)|m;\hat{s}) > \frac{1}{R}$, and she minimizes liquidity injection otherwise. *Q.E.D.*

Proof of Corollary 2:

Proposition 1 implies that $\mathbb{E}(P(\theta)|m;\hat{s})$ decreases in m. It follows directly from Lemma 3. *Q.E.D.*

Proof of Lemma 4:

Following the proof of Lemma 7, we can show that for any $K \in [0, r_1)$, and any $\hat{m} \in (0,1)$, given equations (21) and (22), depositor's incremental payoff from staying until $t = 2$ satisfies single crossing property. The posterior distribution $f(\theta|s_i)$ satisfies monotone likelihood ratio property. Therefore, for any $K \in [0, r_1)$, and any $\hat{m} \in (0,1)$, there exists a unique solution to

$$\int_0^1 \int_{-\infty}^{\infty} \left[c_2\left(m(\lambda,\theta;\hat{s}),\theta;K,\hat{m}\right) - c_1\left(m(\lambda,\theta;\hat{s}),\theta;K,\hat{m}\right)\right] f(\theta|s_i)\,d\theta\, d\lambda = 0 \tag{41}$$

Therefore, there exists a unique solution to $V(\hat{s};K,\hat{s}) = 0$ for any $K \in [0, r_1)$. If $K = r_1$, then for any $\theta \in \mathbb{R}$ and any $m \in [0,1]$, the incremental payoff is non-negative. Staying until $t = 2$ is a strictly dominating strategy, and therefore $\hat{s} = -\infty$.

Recall from Corollary 2 that $\hat{m}^{BR}$ increases in $\hat{s}$. Therefore, deposit insurance K, reducing run incentives can crowd out liquidity injection. However, this effect is secondary, and in equilibrium $\hat{s}$ decreases in K. To see this, suppose the opposite is true – $\hat{s}_1 > \hat{s}_2$ for some $K_1 > K_2$. Then the liquidity injection threshold $m_1^{BR} > m_2^{BR}$ by Corollary 2. However, if $K_1 > K_2$ and $m_1^{BR} > m_2^{BR}$, then it must be the case that $\hat{s}_1 < \hat{s}_2$ because both policies reduce run incentive. Contradiction. Hence, $\hat{s}$ decreases in K. *Q.E.D.*

Proof of Lemma 2:

The policymaker's objective

$$\mathbb{E}\left[u_g^{DI}(\theta,\lambda;\hat{s})\right] = \int_{\underline{\theta}(r_1^{-1};\hat{s})}^{\infty} \int_0^{\frac{\frac{1}{r_1}-n(\theta;\hat{s})}{1-n(\theta;\hat{s})}} \left(1 - r_1\left(\lambda + (1-\lambda)n(\theta;\hat{s})\right)\right) d\lambda \left(p(\theta)R - 1\right)\phi(\theta)d\theta \quad (42)$$

$$= \int_0^{\frac{1}{r_1}} (1 - r_1 m) \int_{\underline{\theta}(m;\hat{s})}^{\infty} \left(p(\theta)R - 1\right) \frac{\phi(\theta)}{1 - n(\theta;\hat{s})} d\theta dm \quad (43)$$

where $n(\theta;\hat{s}) = \Phi\left(\sqrt{\beta}\left(\hat{s}-\theta\right)\right)$ and $\underline{\theta}(m;\hat{s}) = \hat{s} - \sqrt{\beta}^{-1}\Phi^{-1}(m)$. The second equality is derived by changing the variable of integration $m = \lambda + (1-\lambda)n(\theta;\hat{s})$ and then interchanging the order of integration.

Recall the policymaker's inference about bank fundamentals θ conditional on observing the measure of early withdrawal m given by Equation (14). We can express the p.d.f. of the posterior distribution of θ as

$$f(\theta|m;\hat{s}) = \begin{cases} \frac{\frac{\phi(\theta)}{1-n(\theta;\hat{s})}}{f(m;\hat{s})} & \text{if } \theta \geq \underline{\theta}(m;\hat{s}); \\ 0 & \text{otherwise,} \end{cases} \quad (44)$$

where the unconditional distribution of m is

$$f(m;\hat{s}) = \int_{\underline{\theta}(m;\hat{s})}^{\infty} \frac{\phi(\theta)}{1-n(\theta;\hat{s})} d\theta. \quad (45)$$

Therefore, the objective of the policymaker can be alternatively expressed as

$$\mathbb{E}\left[u_g^{DI}(\theta,\lambda;\hat{s})\right] = \int_0^{\frac{1}{r_1}} \int_{\underline{\theta}(m;\hat{s})}^{\infty} \left(p(\theta)R - 1\right) f(\theta|m;\hat{s}) d\theta \left(1 - r_1 m\right) f(m;\hat{s}) dm \quad (46)$$

$$= \mathbb{E}\left[\left(\mathbb{E}\left[p(\theta)|m;\hat{s}\right]R - 1\right)\left(1 - r_1 m\right)|m < r_1^{-1};\hat{s}\right] \quad (47)$$

The first-order derivative of the policymaker's objective in Equation

$$\frac{d}{d\hat{s}}\mathbb{E}\left[u_g^{DI}(\theta,\lambda;\hat{s})\right] = -\sqrt{\beta}r_1 \int_0^{\frac{1}{r_1}} (1-m) \int_{\underline{\theta}(r_1^{-1};\hat{s})}^{\infty} \left(p(\theta)R - 1\right) \frac{\phi(\theta)\phi\left(\sqrt{\beta}\left(\hat{s}-\theta\right)\right)}{\left(1-n(\theta;\hat{s})\right)^2} d\theta dm \quad (48)$$

$$= -r_1 \int_0^{r_1^{-1}} (1-m) \int_0^{r_1^{-1}} \left(p\left(\underline{\theta}(n;\hat{s})\right)R - 1\right) \frac{\phi\left(\underline{\theta}(n;\hat{s})\right)}{(1-n)^2} dndm. \quad (49)$$

In the limit of vanishing information friction, the first-order derivative becomes

$$\lim_{\beta\to\infty} \frac{d}{d\hat{s}}\mathbb{E}\left[u_g^{DI}(\theta,\lambda;\hat{s})\right] = -r_1 \int_0^{r_1^{-1}} (1-m) \int_0^{r_1^{-1}} \left(\frac{1}{1-n}\right)^2 dndm\phi\left(\hat{s}\right)\left(p\left(\hat{s}\right)R - 1\right). \quad (50)$$

Hence, the first-order condition implies that $\lim_{\beta\to\infty} p\left(s^*_{DI}\right)R - 1 = 0$, that is, $\lim_{\beta\to\infty} s^*_{DI} = p^{-1}\left(R^{-1}\right)$. *Q.E.D.*

Proof of Proposition 2:

Since the expression for the policymaker's objective is different depends on the liquidity injection threshold, we discuss the two cases separately.

Case 1: $\hat{m}^{BR} < r_1^{-1}$

In this case, the policymaker implements a small run threshold $\hat{s}$ such that $\hat{m}^B R < r_1^{-1}$, and the policymaker's payoff is given by Equation (26). The policymaker's objective, i.e. her expected payoff, can then be expressed as

$$\mathbb{E}\left[u_g(\theta,\lambda;\hat{s},\hat{m}^{BR})\right] = \mathbb{E}\left[u_g^{DI}(\theta,\lambda;\hat{s})\right] + r_1\int_{\underline{\theta}(\hat{m}^{BR};\hat{s})}^{\infty}\int_0^{\frac{\hat{m}^{BR}-n(\theta;\hat{s})}{1-n(\theta;\hat{s})}} m(\lambda,\theta;\hat{s})d\lambda\left(p(\theta)R-1\right)\phi(\theta)d\theta \tag{51}$$

$$= \mathbb{E}\left[u_g^{DI}(\theta,\lambda;\hat{s})\right] + r_1\int_0^{\hat{m}^{BR}} m\int_{\underline{\theta}(m;\hat{s})}^{\infty}\left(p(\theta)R-1\right)\frac{\phi(\theta)}{1-n(\theta;\hat{s})}d\theta dm \tag{52}$$

where $n(\theta;\hat{s}) = \Phi\left(\sqrt{\beta}\left(\hat{s}-\theta\right)\right)$, $\underline{\theta}(m;\hat{s}) = \hat{s} - \sqrt{\beta}^{-1}\Phi^{-1}(m)$, and $\mathbb{E}\left[u_g^{DI}(\theta,\lambda;\hat{s})\right]$ is the policymaker's objective when she uses only deposit insurance given by Equation (43). Take the first-order derivative of the objective function

$$\frac{d}{d\hat{s}}E\left[u_g(\theta,\lambda;\hat{s},\hat{m}^{BR})\right] = \frac{\partial}{\partial\hat{s}}E\left[u_g(\theta,\lambda;\hat{s},\hat{m}^{BR})\right] + \frac{\partial}{\partial\hat{m}}E\left[u_g(\theta,\lambda;\hat{s},\hat{m})\right]|_{\hat{m}=\hat{m}^{BR}}\frac{d}{d\hat{s}}\hat{m}^{BR} \tag{53}$$

where $\hat{m}^{BR}$ is the optimal liquidity injection threshold given by Equation (19).

We first examine the partial derivative with respect to $\hat{m}$.

$$\frac{\partial}{\partial\hat{m}}E\left[u_g(\theta,\lambda;\hat{s},\hat{m})\right] = \hat{m}r_1\int_{\underline{\theta}(\hat{m};\hat{s})}^{\infty}\left(p(\theta)R-1\right)\frac{\phi(\theta)}{1-n(\theta;\hat{s})}d\theta = \hat{m}r_1\left(\mathbb{E}\left(P(\theta)|\hat{m};\hat{s}\right)R-1\right). \tag{54}$$

Recall from Equation (19) that the optimal liquidity injection threshold is such that the bank's long-term investment has zero NPV, i.e., $\mathbb{E}\left(P(\theta)|\hat{m}^{BR};\hat{s}\right)R - 1 = 0$. Therefore, the second term in Equation (53) is zero.

We then examine the partial derivative with respect to $\hat{s}$. After manipulation, we can express the partial derivative as follows,

$$\frac{\partial}{\partial\hat{s}}\mathbb{E}\left[u_g(\theta,\lambda;\hat{s},\hat{m}^{BR})\right] = -r_1\int_{\hat{m}^{BR}}^{\frac{1}{r_1}}\alpha(m;\hat{s})dm - r_1\hat{m}^{BR}\alpha(\hat{m}^{BR};\hat{s}) \tag{55}$$

where

$$\alpha(m;\hat{s}) = \sqrt{\beta}(1-m)\int_{\underline{\theta}(m;\hat{s})}^{\infty} \frac{(p(\theta)R-1)\,\phi(\theta)}{1-n(\theta;\hat{s})} \frac{\phi\left(\sqrt{\beta}\,(\hat{s}-\theta)\right)}{1-n(\theta;\hat{s})} d\theta. \tag{56}$$

Note that the optimal liquidity injection threshold $\hat{m}^{BR}$ is such that

$$\int_{\underline{\theta}(\hat{m}^{BR};\hat{s})}^{\infty} \frac{(p(\theta)R-1)\,\phi(\theta)}{1-n(\theta;\hat{s})} d\theta = 0. \tag{57}$$

In addition, $\frac{\phi\left(\sqrt{\beta}(\hat{s}-\theta)\right)}{1-n(\theta;\hat{s})}$ strictly decreases in θ. Comparing equations (56) and (57), we have $\alpha(m;\hat{s}) < 0$ for any $m \geq \hat{m}^{BR}$. This implies that $\frac{\partial}{\partial \hat{s}}\mathbb{E}\left[u_g(\theta,\lambda;\hat{s},\hat{m}^{BR})\right] > 0$, i.e. the first term in Equation (53) is positive for any $\hat{s}$ such that $\hat{m}^{BR} < r_1^{-1}$.

Case 2: $\hat{m}^{BR} \geq r_1^{-1}$

In this case, the policymaker's objective can be expressed as

$$\mathbb{E}\left[u_g(\theta,\lambda;\hat{s},\hat{m}^{BR})\right] = \int_{\underline{\theta}(\hat{m}^{BR};\hat{s})}^{\infty} \frac{\hat{m}^{BR}-n(\theta;\hat{s})}{1-n(\theta;\hat{s})}\,(p(\theta)R-1)\,f_\theta(\theta)d\theta. \tag{58}$$

Again, the first-order derivative can be decomposed into two parts as in Equation (53). The partial derivative with respect to $\hat{m}$ is again zero at the optimal liquidity injection threshold $\hat{m}^{BR}$. Specifically,

$$\frac{\partial}{\partial \hat{m}}E\left[u_g(\theta,\lambda;\hat{s},\hat{m})\right] = \int_{\underline{\theta}(\hat{m};\hat{s})}^{\infty} (p(\theta)R-1)\,\frac{\phi(\theta)}{1-n(\theta;\hat{s})}d\theta = \mathbb{E}\left(P(\theta)|\hat{m};\hat{s}\right)R-1. \tag{59}$$

Tthe optimal liquidity injection threshold $\hat{m}^{BR}$ is such that the bank's long-term investment has zero NPV, i.e., $\mathbb{E}\left(P(\theta)|\hat{m}^{BR};\hat{s}\right)R-1=0$. Therefore, the second term in Equation (53) is zero.

We then examine the partial derivative with respect to $\hat{s}$. After manipulation, we can express the partial derivative as

$$\frac{\partial}{\partial \hat{s}}E\left[u_g(\theta,\lambda;\hat{s},\hat{m}^{BR})\right] = -\alpha(\hat{m}^{BR};\hat{s}) > 0. \tag{60}$$

Therefore, the first term in Equation (53) is positive. Hence, the first-order derivative is positive for any $\hat{s}$ such that $\hat{m}^{BR} \geq r_1^{-1}$

Since the first-order derivative of $\hat{s}$ is always positive in both cases, the optimal deposit insurance should achieve the highest $\hat{s}$. In other words, the optimal deposit insurance should allow for as much run as possible. We show in Lemma 4 that depositors' run threshold $\hat{s}$ decreases in deposit insurance policy K. Therefore, the optimal deposit insurance policy should be $K^* = 0$.

Q.E.D.

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