Intervention with Screening in Global Games*

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Abstract

We analyze a canonical binary-action coordination game under the global-games framework. To reduce coordination failure, we propose a novel intervention program that screens agents based on their heterogeneous beliefs of the coordination results. Compared with the conventional government-guarantee type of programs, it incurs a lower cost of implementation and suffers less from moral hazard problems. In equilibrium, only a small mass of "pivotal agents" receiving medium signals self-select to participate in the program. However, the effect is amplified by higher-order beliefs, and coordination failures can be significantly reduced. With negligible information frictions, the proposed program achieves the first-best outcome at zero expected cost. The proposed program can be applied to reduce coordination failures in a wide range of economic contexts.

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1 Introduction

In many economic environments, strategic complementarities among agents can give rise to coordination failure.[1] To reduce the welfare loss from coordination failure, policy makers may intervene by providing incentives for agents to play the socially desirable equilibrium. For instance, during the recent financial crisis, governments around the world provided explicit and implicit guarantees on debt obligations of financial institutions to prevent "runs" on the financial systems. While these policies proved to be effective in restoring financial stability, some drawbacks also emerged. First, implementing guarantee programs at such large scale exposes the policy maker to large costs, which jeopardized sovereign debt sustainability and led to the sovereign debt crisis in many European countries (Acharya, Drechsler and Schnabl, 2014; Farhi and Tirole, 2016). Second, the policies were criticized for their vulnerability to moral hazard problems (Kareken and Wallace, 1978; Keeley, 1990; Cooper and Ross, 2002).[2]

Given that such large-scale interventions are costly, a natural question is whether it is possible to reduce the size of intervention programs without compromising the effectiveness. To answer this question in a general context, we consider a coordination game with incomplete information as in standard global games (Morris and Shin, 2003). When agents receive private signals, they form interim beliefs regarding the expected payoffs from taking different actions. We propose a group of programs with voluntary participation that screen agents based on their interim beliefs. Compared with the conventional government-guarantee type of programs, it has two main advantages. First, in equilibrium, only a small group of marginal investors self-select into the program, which reduces the implementation costs. Also, our proposed programs have the advantage that moral hazard problems are limited to the small group of participating agents.

This paper provides novel insights for the design of intervention policies to reduce coordination failure in various economic contexts. Some existing literature (Sakovics and Steiner, 2012; Choi, 2014) has studied policies that target ex-ante important agents based on their payoff functions. We contribute to this literature by highlighting the role of agents' interim beliefs of the economic fundamental and other agents' actions. If an ex-ante important agent is very optimistic about the coordination result, there's no need to provide extra incentives for her to take the socially desirable action. It is more cost-efficient if the resources are allocated to agents who have medium beliefs and are at the margin of taking the socially

[1] Examples of coordination failures include but not limited to bank run (Diamond and Dybvig, 1983), currency attack (Obstfeld, 1996), macroeconomic coordination failure (Cooper and John, 1988) and technological development (Bresnahan and Trajtenberg, 1995).

[2] Allen et al. (2017) endogenizes the effect of government guarantees on banks' excessive risk taking and shows that guarantees are overall welfare improving even with moral hazard problems.

desirable action.

In our benchmark model, we explore a canonical binary-action coordination game under the global games framework. Global games are useful for linking coordination outcome to the underlying fundamental and determining the unique equilibrium. More importantly, they highlight the strategic interactions of agents with heterogeneous private information. In the model, a continuum of agents is each endowed with an investment opportunity. Their investments feature strategic complementarities. Specifically, the investments are successful if and only if the mass of agents investing exceeds a threshold which decreases in the fundamental of the economy. In addition, each agent receives a noisy private signal of the fundamental and makes inferences about the other agents' investment decisions. The game has a unique equilibrium where all agents follow the same threshold strategy. In terms of welfare, there exists a region of weak fundamentals in which agents do not invest, however, the investments would have been successful if all agents were to invest. Therefore, social welfare will be improved if the policy maker can lower the investment threshold and reduce the coordination failure region. The setup of the model is fairly general such that it can be applied to various economic contexts. In section 6, we discuss three coordination problems and make policy recommendations based on the proposed intervention policy.

Next, we allow the policy maker to offer a subsidy-tax program with voluntary participation to all agents who invest. If an investor accepts the offer, she receives a direct subsidy. In return, she is required to pay tax when the investment is successful. We classified the intervention programs into three categories based on the subsidy-to-tax ratio. If a program is too austere, i.e. has a low subsidy-to-tax ratio, no agents will participate. We call this type of programs the *zero-participation programs*. If a program is too generous such that all investors participate, we call it a *full-participation program*. Many existing intervention policies, including government guarantees and direct subsidies, benefit all agents uniformly and therefore fall into this category. We show that full-participation programs can effectively reduce coordination failure, however, are costly to implement. To reduce the costs of implementation, we propose *partial-participation programs* with medium subsidy-to-tax ratios. A partial participation program is equivalent to a costly insurance policy and screens agents based on their interim belief of success. The most optimistic investors who believe in a high probability of paying the tax do not take the offer. At the same time, the most pessimistic agents who believe in a high probability of coordination failure do not find it worthwhile to invest solely to take advantage of the offer. Only agents with intermediate beliefs will participate in the program since it provides protection against coordination failure and investment loss. We show that with a partial-participation program there is a unique Bayesian Nash equilibrium, in which all agents follow the same threshold strategy with two thresholds. An agent will invest and reject the offer if she receives a high signal; she will invest and accept the offer if her signal is medium and between the two thresholds; she will not invest if she receives a low signal. When the information friction goes to zero, the two thresholds converge, and the expected mass of agents who accept the offer goes to zero, which implies zero expected cost of implementation for the policy maker. Furthermore, with proper choice of subsidy and tax, coordination failures can be eliminated, and the first-best investment threshold can be achieved.

To understand intuitively how partial-participation programs can improve coordination results at a minimal cost, let us start with the original threshold equilibrium without intervention programs. For agents receiving signals right below the investment threshold, without any intervention policy, they will not invest in fear of coordination failure and investment loss. The partial-participation programs provide protection against investment loss and give them extra incentive to make the investment. Therefore, with the partial-participation programs, all agents rationally expect the mass of agents who invest to increase and the strategic complementarities strengthen all agents' incentive to invest. Hence, agents receiving even lower signals would be willing to accept the offer and invest, which is also expected by all agents in the economy and gives them more incentive to invest. Repeating the thought process, the extra incentive to invest provided by the partial-participation programs is amplified by higher-order beliefs, and the investment threshold can be reduced significantly in equilibrium. Given that all agents are more optimistic and less worried about coordination failure, the downside protection of the partial-participation programs becomes less appealing, and the mass of investors who accept the offer in equilibrium is actually small.

We then compare government guarantee programs with partial-participation programs in the presence of moral hazard problems. Government guarantee programs are a special case of full-participation programs and have been widely used to reduce coordination failure. We extend the benchmark model by assuming that after investment, an investor can earn private benefit by shirking, which will reduce the success probability of her own investment. Both types of intervention programs reduce investors' "skin in the game" hence induce shirking at the expense of the policy maker and social welfare. For example, in the context of credit freeze when banks abstain from lending, government guarantees reduce banks' incentive to screen and monitor borrowers. Moral hazard problem critically limits the scale of the government-guarantee type of programs. Specifically, if a government guarantees a large amount of investment losses, all investors, including the most optimistic ones, would participate and shirk. In contrast, for partial-participation programs, the moral hazard problem is limited to the program participants. For the optimistic agents, rejecting the offer and exerting effort gives higher payoff than participating and shirking. Hence, the social welfare loss only incurs for medium-belief agents, the mass of whom goes to zero in the limit of vanishing information frictions. As a result, in the limit, there exist partial-participation programs that can restore the first best, yet no government-guarantee type of programs can restore the first best.

Besides the benchmark model, we also show that the results could be generalized to allow unobservable ex-ante heterogeneity in agents' payoff and information structure. Regarding ex-ante agent heterogeneity, a closely related paper is Sakovics and Steiner (2012). The difference is that they only allow the policy maker to provide direct subsidies conditional on agents' observable heterogeneities. Under their setup, the most cost-efficient subsidies should target the important agents with specific ex-ante characteristics. However, their policy space falls into the category of full-participation programs in our model, and the policy maker can save costs and limit moral hazard problems by switching to a partial-participation program. In other words, we show that subsidization should target the interim rather than ex-ante "pivotal" types. Moreover, since the "pivotal" agents self-select to participate in partial-participation programs, the policy maker does not need to observe agents' ex-ante characteristics. We also show that the binary payoff structure in the baseline model can be generalized to a continuous monotonic payoff function.

Our paper is related to two lines of literature. First, our model is built on the literature of global games which was pioneered by Carlsson and Van Damme (1993). Researchers have applied the global games techniques to analyze coordination failures in different contexts, to name a few, bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005), currency attack (Morris and Shin, 1998), credit freeze (Bebchuk and Goldstein, 2011), debt rollovers (Morris and Shin, 2004; He and Xiong, 2012), and political revolutions (Edmond, 2013). We take a general approach and propose intervention programs that can be applied to reduce coordination failure in different contexts. Morris and Shin (2003) reviews the most commonly applied setup and applications of global games. Our main model in section 2 is a special case with binary payoffs. In section 5, we discuss a generalized payoff structure as in Morris and Shin (2003). In both cases, we show that there exists costless intervention to reduce the coordination threshold and eliminate coordination failures in the limit of zero information friction.

Second, our mechanism shares similar ideas found in the literature that explores policies targeting a specific group of agents to reduce coordination failures. For example, within the contracting literature, Segal (2003) and Bernstein and Winter (2012) show that the optimal policy is to *divide and conquer*, i.e. subsidize a subset of players so that they invest even if no one else invests, then the surplus of players in the no-subsidy set can be fully extracted. Sakovics and Steiner (2012) and Choi (2014) analyzed a coordination game with ex-ante heterogeneous agents and showed that different types should be subsidized in a certain order. These papers all demonstrate that subsidizing a subset of agents to ensure their participation can efficiently encourage the participation of the rest of the agents and reduce coordination failure. Our proposed intervention program is different in terms of implementation. The policy maker offers the same option to all agents, and a subset of agents self-select to participate in the program. In the generalization of unobservable ex-ante heterogeneity, we show that our proposed intervention program is more cost-efficient and does not require information about agents' heterogeneity. Cong, Grenadier and Hu (2017) and Basak and Zhou (2017) analyze intervention policies under dynamic settings. In both papers, the policy maker target a subset of agents in each period. The coordination result of the current period serves as a public signal of the fundamental of the economy. They emphasize the effect of the public signal on agents' beliefs and behaviors in the subsequent period(s). Another closely related paper is Morris and Shadmehr (2017), which analyzes the reward schemes for a revolutionary leader to elicit effort from citizens. The optimal reward scheme also screens citizens for their optimism. However, they consider bounded reward schemes imposed on a continuous and unbounded effort choice set, while we focus on subsidy-tax programs that agents can voluntarily choose to participate in. More importantly, while they assume zero cost for implementing any reward scheme, we target minimizing the cost of intervention.

The rest of the paper is organized as follows. In section 2, we present a benchmark model of a binary-action investment game and introduce intervention policies that can reduce coordination failures. Section 3 and 4 compare the proposed program with government-guarantee type of programs in terms of implementation cost and robustness to moral hazard problems. Two extensions of the benchmark model are discussed in section 5. Section 6 presents several applications of the benchmark model and discusses policy recommendations in each context. Finally, section 7 concludes.

2 The Benchmark Model

In this section, we analyze a binary-action investment game in which each agent's investment outcome depends on the aggregate investment in the economy. In such an environment, inefficient coordination failure can arise in which agents abstain from investment because of their self-fulfilling expectation that other agents will not invest. Then we introduce intervention policies and show how they can encourage investment and reduce coordination failure.

2.1 Setups

There is a unit mass of ex-ante identical infinitesimal agents, indexed by $i \in [0,1]$. These agents are endowed with the same investment opportunity, and they simultaneously make investment decisions $a_i \in \{0,1\}$. $a_i = 1$ if agent i invests, and $a_i = 0$ if agent i does not invest. Not investing results in zero payoffs, while investing incurs a fixed cost $c > 0$ and generates a profit of $b > c$ if agent i's project is successful and 0 if it fails. We assume all agents' investment payoffs are perfectly correlated. The investments would be successful when the fundamentals of the economy are strong enough or a sufficient number of agents invest. Specifically, the payoff from an investment project is

$$\pi(\theta, l) = \begin{cases} b - c, & \text{if } l \geq 1 - \theta, \\ -c, & \text{if } l < 1 - \theta. \end{cases}$$

where $l = \int_0^1 a_i \mathrm{d}i$ represents the fraction of investors or the aggregate investment level, and θ stands for the fundamentals of the economy. Note that agents' investment decisions feature strategic complementarities, because each project is more likely to succeed when more agents choose to invest. When the fundamentals are higher, it requires less aggregate investment to make the projects successful. Without information friction, when $\theta \in [0,1)$, all agents investing ($l = 1$) and all agents not investing ($l = 0$) are both Nash equilibria. However, all agents investing is strictly more efficient than the other equilibrium. Therefore, the first-best outcome is that all agents coordinate to invest when $\theta \geq 0$ and not to invest when $\theta < 0$.

We follow the standard global games setup and assume the following information structure. The fundamental θ is drawn from a uniform distribution with support $[\underline{\theta}, \bar{\theta}]$ and it is not directly observable to the agents when they make investment decisions.[3] Instead, each agent receives a noisy signal about the fundamental $x_i = \theta + \sigma\varepsilon_i$, where ε_i is identically and independently distributed with a continuous and strictly increasing c.d.f. $F(\varepsilon)$, the support of which is $[-\frac{1}{2}, \frac{1}{2}]$. Furthermore, we assume that $\underline{\theta} < -\sigma$ and $\bar{\theta} > 1 + \sigma$. Under this assumption, there exist two dominance regions of signals, $[-\underline{\theta} - \frac{1}{2}\sigma, \underline{x})$ and $(\bar{x}, \bar{\theta} + \frac{1}{2}\sigma]$, with $\underline{x}$ and $\bar{x}$ defined as

$$\Pr(\theta \geq 1 | x = \bar{x}) = \frac{c}{b},$$
$$\Pr(\theta \geq 0 | x = \underline{x}) = \frac{c}{b}.$$

Intuitively, with the lowest aggregate investment level $l = 0$, an agent is indifferent between

[3] We assume a uniform prior to obtain an analytical solution to the coordination game. This is without loss of generality since it can be viewed as a limiting case as the size of the information friction goes to zero.

the two actions when she receives signal $\bar{x}$. Therefore, her dominant strategy when signal $x > \bar{x}$ is to invest. Similarly, with the highest aggregate investment level $l = 1$, an agent is indifferent between the two actions if she observes signal $\underline{x}$. Hence, when $x < \underline{x}$, not investing is the dominant strategy.

2.2 Equilibrium without Intervention

In this subsection, we analyze the equilibrium without intervention and identify the inefficiencies due to coordination failure. Proposition 1 characterizes the equilibrium.

Proposition 1 *Without intervention, there is a unique equilibrium in which all agents follow the same strategy*

$$a_i(x_i) = \begin{cases} 1, & \text{if } x_i \geq \xi_0^*, \\ 0, & \text{if } x_i < \xi_0^*. \end{cases}$$

where $\xi_0^* = \frac{c}{b} + \sigma F^{-1}\left(\frac{c}{b}\right)$.

Since there is a continuum of agents, given the realization of fundamentals θ, we can apply the law of large numbers to calculate the aggregate investment l and predict the coordination outcomes. In equilibrium, all agents follow the same threshold strategy. Therefore, the coordination outcome also has a threshold above which the investment projects are successful. Let $\theta^*(\xi)$ denote the fundamental threshold when all agents follow the threshold strategy ξ, then it is defined by

$$F\left(\frac{\theta^*(\xi) - \xi}{\sigma}\right) = 1 - \theta^*(\xi).$$

In words, at the fundamental threshold, the fraction of investors l equals the cutoff $1 - \theta$. Then the fundamental threshold in equilibrium is given by

$$\theta^*(\xi_0^*) = \frac{c}{b}.$$

The fundamental realizations can be divided into three regions as shown below. In the middle

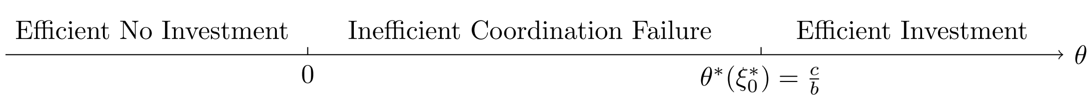


Figure 1: Coordination Results

region $\theta \in \left[0, \frac{c}{b}\right)$, if all agents coordinate to invest, the investment projects would have been successful. However, the agents have self-fulfilling beliefs that other agents do not invest. As a result, they rationally choose not to invest. Since a unit of successful investment generates a positive surplus of $b - c$, in the middle region, coordination failure leads to social welfare loss of $b - c$. Hence, the first-best scenario has a fundamental threshold θ^* equal to zero. And in the next section, we will show how our proposed intervention program can lower this cutoff and reduce inefficiencies caused by coordination failure.

2.3 Intervention Program

Having characterized the equilibrium in the game without intervention, we now describe the subsidy-tax intervention program that the policy maker can use to boost investment and reduce coordination failure. The intervention program consists of two parts, a direct subsidy $s \in [0, c]$ and a contingent tax $t \in [0, b)$. Specifically, if an investor decides to accept the offer, she receives an upfront subsidy s regardless of the investment outcome and pays a lump-sum tax t only if the investment succeeds.[4] The program is only available to the investors and they voluntarily decide whether to participate in the program. Note that there is an implicit assumption that the actions taken by the agents are observable to the policy maker and can be contracted on. We make this assumption because, as shown in Bond and Pande (2007), if the policy maker cannot observe individual actions, its ability to use subsidy-tax schemes as a coordination device is greatly limited. This assumption imposes certain limitations on the application of our proposed intervention mechanism. For example, in the context of currency attack, it is hard to trace agents' action and tax conditional on agents' investment behavior. Therefore, the intervention program discussed in this paper cannot be applied to solving currency deflation caused by coordination failure (Morris and Shin, 1998). Despite this limitation, there is a wide range of real-world applications. In section 6, we discuss three representative examples.

Mathematically, if an investor accepts the offer, her payoff is modified to

$$\tilde{\pi}(\theta, l) = \begin{cases} b - t - (c - s), & \text{if } l \geq 1 - \theta, \\ -(c - s), & \text{if } l < 1 - \theta. \end{cases}$$

The upfront subsidy s reduces the cost of investment and encourages agents to invest. The contingent tax t directly helps the policy maker recover the cost of providing subsidies. More importantly, it will become clear later that the contingent tax t indirectly saves cost by deterring participation of optimistic agents. The timeline of the coordination game with the

[4] Since in the benchmark model there's only two possible payoffs from investing, we only need to specify a contingent lump-sum tax. In section 5.2, we analyze a more general setup where there's a continuum of investment outcomes and we allow tax to be proportional to the investment revenue.

intervention program is modified as follows. At the beginning of the game, the policy maker announces the intervention program (s, t). Note that since the subsidy s and tax t are both state independent, the announcement of the intervention program does not convey any information possessed by the policy maker. Angeletos, Hellwig and Pavan (2003) demonstrates that the informational role of state contingent policy can lead to multiple equilibria in global games. Therefore, the intervention programs analyzed in this paper are free from the signaling concern of state contingent policies and do not require the policy maker to have superior information about the fundamentals of the economy. Then the fundamental θ is realized, and each agent receives a noisy signal of the fundamental. After observing the signal, agents simultaneously make their decisions on whether to invest and if so, whether to participate in the intervention program. As soon as the decisions are made, active investors pay the cost c, and the policy maker transfers the subsidy s to all investors participating in the intervention program. Then the aggregate investment l and the investment returns are realized. Finally, the policy maker collects tax t from the investors participating in the intervention program if the investments are successful. The timeline is summarized in Figure 2 below.

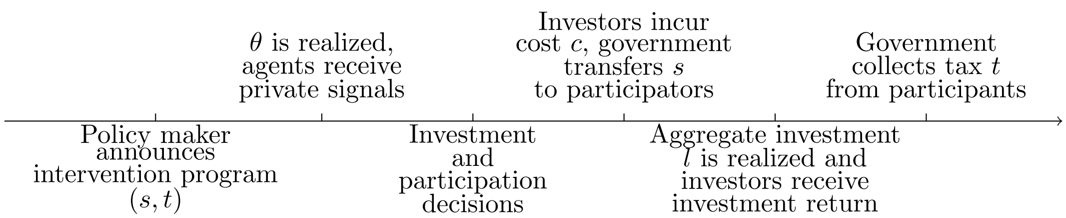


Figure 2: Timeline of the Investment Game

Although the intervention program is specified as a subsidy-tax program, it can be interpreted as other forms of intervention with transfers between the policy maker and the investors, contingent on the coordination result. For example, a government-guarantee type program that promises to cover the loss of failed investment up to $s^g \leq c$ is equivalent to a subsidy-tax program with $s = t = s^g$. To see this, under both programs, the net transfer from the government to any participating investor is 0 in the case of successful investments and s^g in the case of failed investments. Similarly, an asset purchase program in which the policy maker buys $\frac{t}{b}$ fraction of the project with price s is equivalent to a subsidy-tax program (s, t).

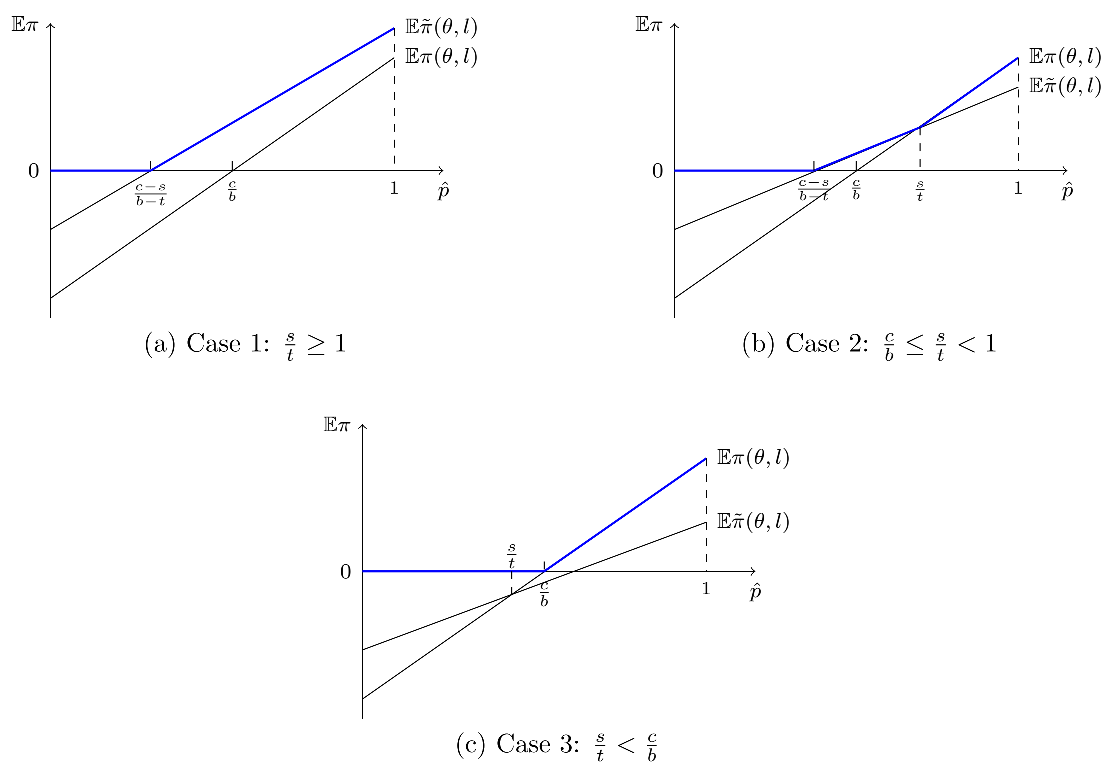
2.4 Equilibrium with Intervention

We now analyze the equilibrium with intervention and demonstrate how the intervention program works to reduce coordination failure. With the intervention program, an agent has three choices: $\{a = 1, Reject\}$, $\{a = 1, Accept\}$, and $\{a = 0\}$. Note that although agents make two decisions, whether to invest and conditional on investing, whether to accept the offer, only their investment decisions affect the coordination results. Therefore, an agent only cares about the investment decisions of the others but not their participation in the intervention program. As a result, to analyze the best response and equilibrium strategies, it is sufficient to condition on other agents' investment strategies. Let $\hat{p}_i = \Pr[l \geq 1 - \theta | x_i]$ denote the interim belief of success of agent i given her private signal x_i and other agents' investment strategies $a_{-i}(x)$. The expected payoffs from $\{a = 1, Reject\}$ and $\{a = 1, Accept\}$ are

$$\mathbb{E}[\pi(\theta, l)|x_i] = \hat{p}_i b - c, \tag{1}$$

$$\mathbb{E}[\tilde{\pi}(\theta, l)|x_i] = \hat{p}_i (b - t) - (c - s) \tag{2}$$

respectively. And the expected payoff from $\{a = 0\}$ is zero. Figure 3 depicts the expected payoff as a function of the interim belief $\hat{p}$. It can be divided into three cases according to the subsidy-tax ratio $\frac{s}{t}$. In the first case when $\frac{s}{t} \geq 1$, accepting the offer dominates rejecting



(a) Case 1: $\frac{s}{t} \geq 1$

(b) Case 2: $\frac{c}{b} \leq \frac{s}{t} < 1$

(c) Case 3: $\frac{s}{t} < \frac{c}{b}$

Figure 3: Expected Payoffs and Interim Beliefs

the offer. This is because investors always receive a higher subsidy s than their tax payment required by the intervention program. We call this type of programs the full-participation programs. Without intervention, the belief threshold for investment is the cost-benefit ratio $\frac{c}{b}$. With a full-participation program, the threshold is lowered to $\frac{c-s}{b-t}$. In the third case with $\frac{s}{t} < \frac{c}{b}$, rejecting dominates accepting the offer. We call this type of programs the zero-participation programs. Thus, the threshold belief under the zero-participation program is the same as the original cost-benefit ratio $\frac{c}{b}$. The second case is the most interesting. When $\frac{c}{b} \leq \frac{s}{t} < 1$ (figure 3.b), an agent would only accept the offer and invest when she has an intermediate belief $\hat{p} \in [\frac{c-s}{b-t}, \frac{s}{t}]$. We call this type of programs the partial-participation programs. Notice in both case 1 and case 2, the provision of the intervention program lowers the threshold belief to $\frac{c-s}{b-t}$. The difference is that, in case 2, the most optimistic agents do not participate in the intervention program, which is cost saving especially when the information friction is small. We will analyze the cost of the programs in detail in section 3.

Next we sketch the analyses of equilibrium with intervention. It will become clear later that iterated deletion of dominated strategies allows us to focus on cutoff investment strategies. We say an agent follows a cutoff investment strategy with threshold k, if her investment strategy is

$$a_i(x;k) = \begin{cases} 1, & \text{if } x \geq k, \\ 0, & \text{if } x < k. \end{cases} \tag{3}$$

Let $p(x;k)$ denote the interim belief of success when an agent receives private signal x and all other agents follow a cutoff investment strategy k,

$$p(x;k) = Pr(\theta > \theta^*(k)|x) = F\left(\frac{x - \theta^*(k)}{\sigma}\right), \tag{4}$$

where $\theta^*(k)$ is the fundamental threshold for successful investment and satisfies $F\left(\frac{k-\theta^*(k)}{\sigma}\right) = \theta^*(k)$. An agent's interim belief of success $p(x;k)$ increases in x and decreases in k, because a high private signal x indicates a high realization of fundamentals θ, and a low investment threshold k implies a high aggregate investment l. Both imply a high probability of success.

In all three cases depicted in figure 3, the optimal investment strategy is that an agent invests if and only if her belief $p(x,k)$ exceeds a threshold. Since $p(x,k)$ is monotonic in both x and k, an agent's best response to other agents' cutoff strategy k is also a cutoff investment strategy based on her own signal. The two dominance regions form two extreme cutoff investment strategies. Starting there, by iterated deletion of dominated strategies, we are able to prove the uniqueness of the equilibrium with intervention. The details of the analyses can be found in the proof of proposition 2 below. The following proposition characterizes the equilibrium with a subsidy-tax intervention program (s, t).[5]

Proposition 2 *When the policy maker offers a subsidy-tax intervention program $(s, t) \gg 0$, the game has a unique equilibrium. There are three different cases,*

1. *When $\frac{s}{t} \geq 1$, the equilibrium is for any agent i,*

$$\begin{aligned} a_i = 1, \textit{Accept}, \quad \textit{if} \quad x_i &\geq \xi^*(s,t), \\ a_i = 0, \quad \textit{if} \quad x_i &< \xi^*(s,t). \end{aligned}$$

   where

$$\xi^*(s,t) = \frac{c-s}{b-t} + \sigma F^{-1}\left(\frac{c-s}{b-t}\right),$$

2. *When $\frac{c}{b} \leq \frac{s}{t} < 1$, the equilibrium is for any agent i,*

$$\begin{aligned} a_i = 1, \textit{Reject}, \quad &\textit{if} \quad x_i \geq \eta^*(s,t), \\ a_i = 1, \textit{Accept}, \quad &\textit{if} \quad \xi^*(s,t) \leq x_i < \eta^*(s,t), \\ a_i = 0, \quad &\textit{if} \quad x_i < \xi^*(s,t), \end{aligned}$$

   where

$$\begin{aligned} \xi^*(s,t) &= \frac{c-s}{b-t} + \sigma F^{-1}\left(\frac{c-s}{b-t}\right), \\ \eta^*(s,t) &= \frac{c-s}{b-t} + \sigma F^{-1}\left(\frac{s}{t}\right). \end{aligned}$$

3. *When $\frac{s}{t} < \frac{c}{b}$, the equilibrium is for any agent i,*

$$\begin{aligned} a_i = 1, \textit{Reject}, \quad \textit{if} \quad x_i &\geq \xi^*(s,t), \\ a_i = 0, \quad \textit{if} \quad x_i &< \xi^*(s,t), \end{aligned}$$

   where

$$\xi^*(s,t) = \frac{c}{b} + \sigma F^{-1}\left(\frac{c}{b}\right).$$

[5] Frankel, Morris and Pauzner (2003) prove existence, uniqueness and monotonicity in multi-action global games. However, our setup does not satisfy the continuity assumption. Therefore, we provide our own proof in the Appendix.

The ratio of the upfront subsidy s and the ex-post tax t can be interpreted as the generosity of the program. If the offer is generous (case 1), all investors find it profitable to accept the offer and the equilibrium investment cutoff depends on the modified cost $c' = c - s$ and benefit $b' = b - t$. If the offer is austere (case 3), all investors will not be interested in the offer. Therefore the equilibrium investment cutoff is the same as the original cutoff without the intervention program. The most interesting case is case 2, in which the generosity of the offer is medium. Investors with high private signals have strong beliefs in the success of the project, so they will reject the subsidy offer since they believe in a high probability of paying a net tax in the future. However, even without subsidies, these optimistic agents would invest anyway. Agents with low private signals have strong beliefs in the failure of the project, so even with the subsidy s, they still suffer a loss of $c - s$ from investing. Therefore, these agents would not invest regardless of the intervention program. In contrast, investors receiving signals around the threshold do not have strong beliefs about the coordination results. Without the intervention program, some of these agents would not invest. The intervention program provides insurance against losses in case of failed investment and gives these agents extra incentive to invest. With the extra incentive, these agents' decisions are effectively altered and the aggregate action l therefore increases. The increase in l, in turn, strengthens all agents' incentive to invest. Agents with even lower signals would participate in the program and change their decisions to invest. Through iterations of higher-order beliefs, the action cutoff is significantly lowered. Moreover, agents with signals around the old cutoff are significantly more optimistic, and therefore the intervention program is no longer appealing to them. In equilibrium, the mass of investors accepting the offer is rather small. We call these investors the "pivotal" investors, since the equilibrium investment cutoff is determined by their modified cost and benefit.

In case 1 and 2, the fundamental cutoff above which the investment projects are successful is

$$\theta^*(\xi^*(s,t)) = \frac{c-s}{b-t}. \tag{5}$$

Note that the new fundamental cutoff is lower than that without government intervention. Therefore, the provision of the intervention program successfully reduces the inefficient coordination failure region. If the government picks $s = c$ and $t \in [s, b)$, the fundamental cutoff can be reduced to 0, eliminating the whole region of inefficient coordination failure as demonstrated in Figure 4.

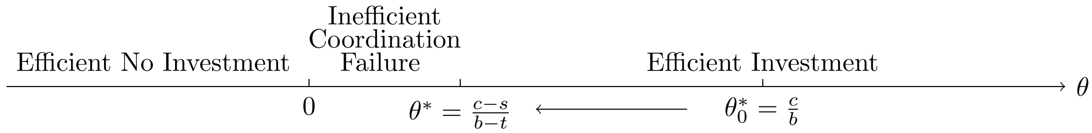


Figure 4: Coordination Results after Intervention

3 Cost of Implementation

In this section, we compare the implementation cost of partial-participation and full-participation intervention programs in two cases, one with negligible information frictions and one with non-negligible information frictions. We then discuss the intuitions why partial-participation is more cost-efficient than full-participation programs.

3.1 Cost of the Intervention Programs

We compare the expected cost of the partial-participation and full-participation programs conditional on the same target fundamental threshold θ^* of successful investment. To allow for the possibility that the policy maker values tax and subsidy differently, the value of tax for the policy maker is normalized to 1 and the cost of providing subsidy is assumed to be τ. The ex-post cost of providing the intervention program to an individual investor is

$$\hat{c}(\theta, s, t) = \begin{cases} \tau s - t, & \text{if } l \geq 1 - \theta, \\ \tau s, & \text{if } l < 1 - \theta. \end{cases} \tag{6}$$

When $\hat{c}(\theta, s, t)$ is negative, the policy maker profits from providing this intervention program.

For the rest of the analyses, we focus on $\tau \geq 1$ for two reasons. First, we believe it is a realistic characterization. If subsidy is provided before tax collection, $\tau > 1$ reflects the funding cost of the policy maker due to the opportunity cost of other welfare-improving programs. Alternatively, if the program is government guarantee, $\tau > 1$ reflects the cost of commitment, such as setting aside funds specifically for the program. Moreover, any administrative cost incurred by providing subsidy or collecting tax can raise τ. Secondly, if $\tau < 1$, given negligible information frictions, the policy maker can easily restore the first best and profit at the same time by offering $t = s = c$.[6] The coordination problem then becomes trivial. Therefore, for the rest of the paper, we assume $\tau \geq 1$.

Let $C(\theta, s, t; \sigma)$ denote the ex-post total cost of providing a subsidy-tax intervention

[6]To be precise, the policy maker should set $t = s = c - \varepsilon$ with a very small ε to avoid over-investment when $\theta < 0$ and keep the left dominance region.

program (s,t) given the realized fundamental θ and the information friction σ. For full-participation programs, i.e., $\frac{s}{t} \geq 1$, all investors participate in the intervention program. The ex-post cost of implementation is

$$C(\theta, s, t; \sigma) = \begin{cases} (\tau s - t)\left[1 - F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right)\right], & \text{if } \theta \geq \frac{c-s}{b-t}, \\ \tau s\left[1 - F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right)\right], & \text{if } \theta < \frac{c-s}{b-t}. \end{cases} \tag{7}$$

For partial-participation programs $\frac{c}{b} \leq \frac{s}{t} < 1$, only pivotal investors participate in the intervention program. In this case,

$$C(\theta, s, t; \sigma) = \begin{cases} (\tau s - t)\left[F\left(\frac{\eta^*(s,t)-\theta}{\sigma}\right) - F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right)\right], & \theta \geq \frac{c-s}{b-t}, \\ \tau s\left[F\left(\frac{\eta^*(s,t)-\theta}{\sigma}\right) - F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right)\right], & \theta < \frac{c-s}{b-t}. \end{cases} \tag{8}$$

If $\frac{s}{t} < \frac{c}{b}$, no agents will find it profitable to opt in to the intervention program, therefore $C(\theta, s, t; \sigma) = 0$.

Proposition 3 below compares the ex-post and ex-ante expected cost of partial-participation programs and full-participation programs, which restore first best in the limit of vanishing information frictions.

Proposition 3 *With strictly costly subsidy $\tau > 1$, when the information friction σ goes to 0, there exists a continuum of full-participation programs (s,t) and a continuum of partial-participation programs (s', t') achieving the first-best outcome, where $s = s' = c$ and $t \leq c < t' \leq b$.*

For any such (s,t) and (s',t'), given θ, the full-participation program (s,t) is ex-post more costly than the partial-participation program (s',t'). Specifically,

$$\begin{aligned} \lim_{\sigma\to 0} C(\theta, s, t; \sigma) = \tau s - t &> \lim_{\sigma\to 0} C(\theta, s', t'; \sigma) = 0, && \text{if } \theta > 0; \\ \lim_{\sigma\to 0} C(\theta, s, t; \sigma) = \tau s - t &> \lim_{\sigma\to 0} C(\theta, s', t'; \sigma) = \tfrac{s'}{t'}(\tau s' - t'), && \text{if } \theta = 0; \\ \lim_{\sigma\to 0} C(\theta, s, t; \sigma) &= \lim_{\sigma\to 0} C(\theta, s', t'; \sigma) = 0, && \text{if } \theta < 0. \end{aligned}$$

Moreover, the full-participation program (s,t) is ex-ante strictly more costly than the partial-participation program (s',t'). Specifically, $\lim_{\sigma\to 0} \mathbb{E}_\theta C(\theta, s, t; \sigma) > \lim_{\sigma\to 0} \mathbb{E}_\theta C(\theta, s', t'; \sigma) = 0$.

The proof is in the Appendices. When the information friction is small, although both full-participation programs and partial-participation programs can effectively reduce coordination failures and restore the first-best outcome, the partial-participation programs are ex-post weakly less costly than the full-participation programs in all states. Intuitively, compared with full-participation programs, partial-participation programs have less participants since the optimistic investors are deterred from participating. This subsequently reduces the cost of implementation. If the policy maker evaluates the ex-ante expected cost of the programs, in the limit of negligible information frictions, the partial-participation programs incur zero cost and strictly dominate the full-participation programs.

Now we extend the analysis to the case of non-negligible information frictions $\sigma > 0$. To facilitate the comparison of the cost of different programs given the same fundamental θ^*, we introduce an alternative parameterization of the intervention programs. Specifically, an intervention program (s, t) can be equivalently parameterized by (θ^*, λ) as follows,

$$s = \frac{c - \theta^* b}{1 - \theta^*} + \theta^* \lambda,$$
$$t = \frac{c - \theta^* b}{1 - \theta^*} + \lambda.$$

$\theta^* = \frac{c-s}{b-t}$ is the target fundamental threshold, and $\lambda = \frac{t-s}{1-\theta^*}$ is proportional to the net tax charged by the program when the project succeeds. λ can also be interpreted as the scale of the program because given the same target θ^*, both tax and subsidy are strictly increasing in λ. Intuitively, when λ increases, the intervention program charges a higher net tax and becomes less attractive. To achieve the same target, the government needs to increase the direct subsidy s (and the tax t at the same time) to provide more downside protection to the investors. When $\lambda \in [-\frac{c-\theta^* b}{1-\theta^*}, 0]$, the subsidy-to-tax ratio $\frac{s}{t} \geq 1$, and the program is a full-participation program. When $\lambda \in (0, \frac{b-c}{1-\theta^*})$, $\frac{c}{b} < \frac{s}{t} < 1$, and the program is a partial-participation program. Note that to achieve the same target θ^*, the partial-participation programs are larger in scale λ than the full-participation programs. The reason is that partial-participation programs are less generous, i.e. have lower subsidy-to-tax ratio than full-participation programs, the magnitude of partial-participation programs need to be larger to provide more downside protection as compensation.

Suppose the policy maker is considering switching from a full-participation program (θ^*, λ) to a partial-participation program (θ^*, λ'). The change in the expected cost of implementation comes from both the extensive and the intensive margin. On the extensive margin, the most optimistic investors will no longer enter the program. Hence, this effect always reduces the expected cost of intervention. However, on the intensive margin, the cost of providing the program to an individual investor could increase or decrease. Formally, the difference in unit cost is

$$\hat{c}(\theta, s', t') - \hat{c}(\theta, s, t) = \begin{cases} (\tau\theta^* - 1)(\lambda' - \lambda), & \text{if } \theta \geq \theta^*, \\ \tau\theta^*(\lambda' - \lambda), & \text{if } \theta < \theta^*. \end{cases}$$

With vanishing information frictions, the effect on the intensive margin is negligible because the mass of participants in partial-participation programs goes to zero except for the knife-edge case of $\theta = \theta^*$. Therefore, switching to any partial-participation program will always reduce the cost of implementation. This is no longer true with non-negligible information frictions. In proposition 4, we provide two sufficient conditions such that switching to a partial-participation program reduces the expected cost of implementation.

Proposition 4 *For any $\sigma > 0$, if $1 \leq \tau < G(\theta^*, 1)$ or $\theta^*(1+\sigma) < 1$, there exists a partial-participation program (θ^*, λ) which achieves θ^* at lower expected cost than any full-participation program targeting θ^*, where $G(\alpha, \beta)$ is defined for any $0 \leq \alpha \leq \beta \leq 1$ as*

$$G(\alpha, \beta) = \frac{\int_{F^{-1}(\alpha)}^{F^{-1}(\beta)} F(x)\mathrm{d}x}{\alpha(F^{-1}(\beta) - F^{-1}(\alpha))}.$$

The proof involves technical details and is included in the Appendix. Here we provide some intuitions. Since partial-participation programs provide more subsidy and charges more tax to each participants, the effect on the intensive margin depends on the ratio of expected mass of taxpayers to the expected mass of subsidy receivers. This ratio is equal to $G(\theta^*, 1)$ in a partial-participation program (θ^*, λ) when λ approaches 0. If $\tau < G(\theta^*, 1)$, the ratio is large enough such that the increase in expected tax revenue is greater than the increase in expected subsidy provision. Hence, the effect on the intensive margin also works in favor of the partial-participation programs, and switching to a partial-participation program with small λ reduces the expected cost. Notice for any given $\theta^* < 1$, $G(\theta^*, 1) > 1$. Therefore, the special case of $\tau = 1$ always satisfies the first condition. The second condition governs the relative importance of the two margins. If θ^* and σ are jointly small, the participation threshold η^* for partial-participation programs is also small, therefore the mass of participants is significantly reduced. In particular, if the second condition holds, the effect on the extensive margin dominates that on the intensive margin, making the proposed partial-participation program less costly than any full-participation programs. In summary, Proposition 4 gives three circumstances in which the most cost-efficient subsidy-tax program is a partial-participation program: ambitious target (small θ^*), small information frictions (small σ), or small cost of subsidy τ. Note that as a special case, if the policy maker targets at the first-best $\theta^* = 0$, there always exists a partial-participation program that dominates all full-participation programs.

We use a numerical example to demonstrates how switching to a partial-participation program from a full-participation program can reduce the expected cost of the intervention.

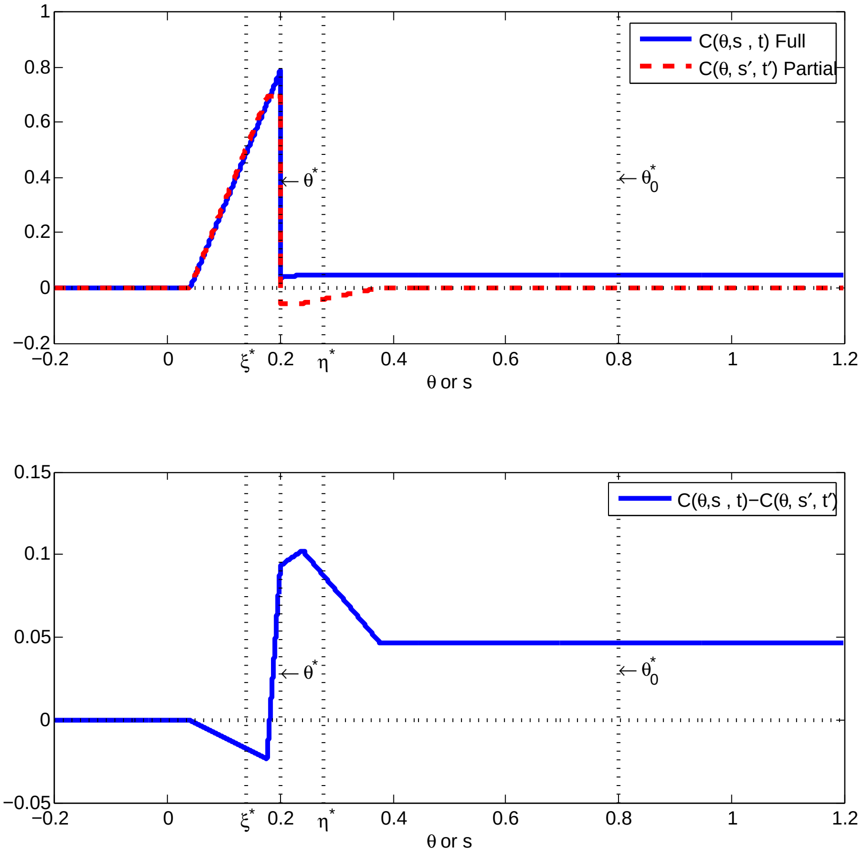


Figure 5: Cost Functions

Suppose the prior on θ is uniformly distributed on $[\underline{\theta}, \bar{\theta}] = [-0.2, 1.2]$. The private noise ε follows a uniform distribution over $[-\frac{1}{2}, \frac{1}{2}]$ and $\sigma = 0.2$. c and b are set to 1 and 1.25, so the benchmark success threshold is $\frac{c}{b} = 0.8$. The policy maker has a cost parameter $\tau = 1.05$ and targets a success threshold $\theta^* = 0.2$. The least costly full-participation program to achieve the equilibrium threshold is $s = t = 0.9375$. The ex-post cost as a function of the realized fundamental is represented by the solid blue line in Figure 5. The cost is positive for all $\theta > \theta^*$ because all investing agents sign up for the program and there's a positive cost $\tau s - t$ of providing this program to each agent. When θ falls below θ^*, the cost surges because the investment projects fail and the policy maker can't recover t. Now the policy maker switches to a partial-participation program. There's a continuum of partial-participation programs that targets the same threshold θ^*. We take $(s', t') = (0.97, 1.1)$ for an example. The red dashed line in the top panel of Figure 5 represents the ex-post cost function of program (s', t'). It has a similar shape as the cost function of the full-participation program. However, it converges to 0 when θ is large enough so that all agents receive signals higher than η^* and no agents participate in the intervention program. The difference between the two cost functions is plotted in the bottom panel. Compared to the full-participation program, the partial-participation program incurs lower cost when $\theta > \theta^*$ because of the higher tax charge and the lower participation rate. When $\theta < \theta^*$, since the partial-participation program provides higher subsidy, it incurs higher cost than the full-participation program. On average, the partial-participation program incurs lower expected cost.

3.2 Discussions

From previous analyses, we show that partial-participation intervention programs can improve the coordination results to the first-best outcome in the investment game, yet has zero cost when the information friction vanishes. This result seems striking at first glance. The most important reason why the partial-participation intervention program works effectively at a minimal cost is that it targets precisely the marginal agents who are on the investment threshold and can be incentivized to invest relatively easily. These agents are also the "pivotal" investors whose investment decisions are crucial in the determination of the investment threshold. The figure below demonstrates how through higher-order beliefs, our proposal effectively reduces coordination failure.

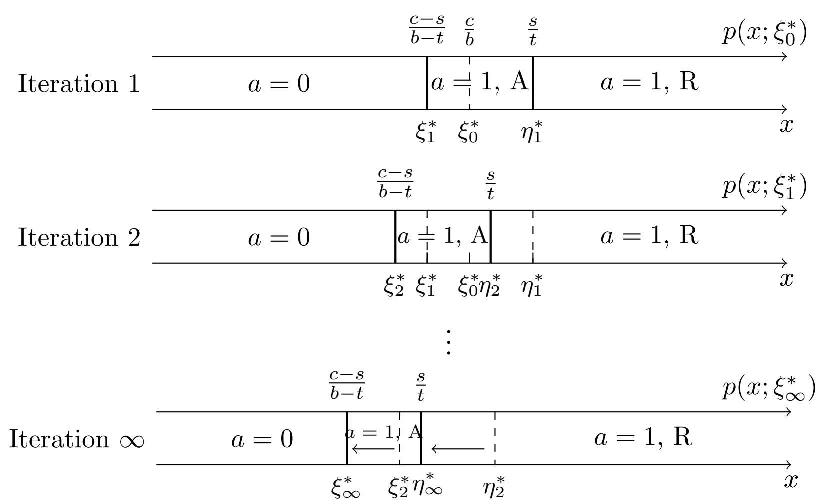


Figure 6: Role of Higher-Order Beliefs

In each iteration, the lower axis denotes the signal received by an agent, and the upper axis denotes the corresponding belief. Start from the cutoff strategy ξ_0^*, which is the original cutoff without intervention. The partial intervention program incentivizes agents to lower the investment threshold to ξ_1^*. Since all agents understand that more agents are willing to invest, given the same private signals, they all believe in a higher aggregate action l and a higher probability of successful investment $p(x;\xi_1^*)$. Therefore, they are willing to lower their investment threshold further to ξ_2^*. Similarly, with the additional mass of agents receiving signals between ξ_1^* and ξ_2^* investing, all agents are more optimistic about the success of the investment and therefore further lower their investment threshold to ξ_3. At the same time, as the agents become more optimistic about their investments, the intervention program becomes less attractive, which implies a decreasing sequence of participation thresholds η_n^*. With an infinite number of iterations, both the investment threshold and the participation threshold are significantly lowered. As the information friction decreases, investors become more certain about the coordination results, so the mass of "pivotal" investors shrinks to zero. However, as long as there exist a few pivotal investors, the intervention program will have a significant effect on the investment threshold due to higher-order beliefs.

Our partial-participation programs share similar spirit to the targeted intervention programs. Sakovics and Steiner (2012) analyze coordination games with heterogeneous agents and argue that the optimal subsidy schedule is to target a certain type of agent. In section 5, we examine an extension with heterogeneous agents and show that there exist partial-participation programs that incur zero cost to restore first-best outcome in the limit of negligible information frictions. Similar to the main model, in equilibrium, only a small mass of "pivotal" agents self-select to accept the policy maker's offer. The only difference is that different agent types have different thresholds, and the "pivotal" agents are the ones receiving signals around their own thresholds. The result conveys one message contrasting Sakovics and Steiner (2012) that policy makers should target interim rather than ex-ante important types. Also, one common problem with targeted intervention programs is that information acquisition to identify the targeted type(s) can be costly. The policy maker needs to correctly identify each agent's type to implement the targeted intervention programs. In contrast, our proposed intervention programs incentivize the "pivotal" agents to self-reveal their types, therefore the implementation only requires information on the payoff structure of different types. As a result, our proposed program is superior to the targeted intervention programs in terms of reducing the costs of collecting information.

4 Interventions in the Presence of Moral Hazard

In this section, we address the concern of moral hazard problem of government guarantees and demonstrate our proposal's robustness to moral hazard problems. For example, in the context of self-fulfilling credit freeze (Bebchuk and Goldstein, 2011), banks may abstain from lending in fear that the other banks will withdraw lending, which results in a coordination failure of credit crunch. If the government provides guarantees on bank losses, the banks may have the incentive to shirk in screening and monitoring the borrowers, since the losses caused by shirking is guaranteed by the government. In order to do incorporate moral hazard problems in the model, we modify the game into two stages. The first stage is the same as the benchmark model with an intervention program, except that the payoffs are not realized until the second stage. If the realized fundamental $\theta < 1 - l$, we say the aggregate state is *Bad*. In this case, the investment project fails and the game ends immediately. If the realized fundamental $\theta \geq 1 - l$, we say the aggregate state is *Good*. In this case, the game enters the second stage, in which investors make their effort choices. If an investor exerts effort, the investor pays a cost of effort c^e, and her project succeeds with probability 1.[7] On the other hand, if an investor shirks, her own project succeeds with probability $1 - \gamma$. As in the benchmark model, the project generates b in case of success and 0 in case of failure. And for the participants in the intervention program, they are required to pay tax t if their investments are successful. We make the following assumption on the parameters.

Assumption 1 *The investment opportunity has the following properties,*

a) shirking is inefficient, $c^e < \gamma b$*;*

b) the investment projects are ex-ante efficient, $b > c + c^e$*.*

Given the assumptions above, the first-best scenario is that all agents invest and exert effort if the fundamental $\theta \geq 0$, and all agents do not invest otherwise.

The equilibrium with moral hazard problem can be solved backward. In the second stage, an investor would exert effort if and only if

$$b - t - c^e \geq (1 - \gamma)(b - t). \tag{9}$$

This condition can be interpreted as a constraint on the size of the tax t,

$$t \leq b - \frac{c^e}{\gamma}. \tag{10}$$

When the tax is above the threshold, participating investors has too little "skin in the game" to exert effort, resulting in inefficient outcomes. Intuitively, with a higher cost of effort c^e or lower losses caused by shirking γ, the incentive problem is more severe, imposing a tighter constraint on the size of tax t.

[7] The results hold as long as the success probability when exerting effort is between $1 - \gamma$ and 1, which prevents the policy maker from inferring effort choice based on ex-post investment outcome. Otherwise, the moral hazard problem can potentially be solved by imposing ex-post punishment when the policy maker observes failed investment.

Next, we will analyze the equilibrium under different programs and examine whether a full-participation program like government guarantee or a partial-participation program can achieve first best when there is moral hazard problem in the private investment project. In the context of our model, we interpret the government guarantee program as a subsidy-tax program (s, t) with $s = t$, which is the full-participation programs with least cost. Since participating in the government guarantee program weakly dominates investing alone, every investor will take advantage of this program.

Government Guarantee. The moral hazard problem in the second stage imposes an upper limit on the scale of the government guarantee program if the policy maker wants to enforce effort.

The expected payoff from investing with the government guarantee program is

$$\mathbb{E}[\tilde{\pi}(\theta, l)|x_i] = \begin{cases} \hat{p}_i(b - t - c^e) - (c - s), & \text{if } t \leq b - \frac{c^e}{\gamma}, \\ \hat{p}_i(1-\gamma)(b - t) - (c - s), & \text{if } t > b - \frac{c^e}{\gamma}. \end{cases} \tag{11}$$

From the analysis of the benchmark model, we know that in the unique Bayesian Nash equilibrium, the fundamental threshold above which the aggregate state is good is equal to the belief of the marginal investor. Given a program with $t \leq b - c^e/\gamma$ that prevents shirking, the fundamental threshold in equilibrium is

$$\theta^* = \frac{c - s}{b - t - c^e}. \tag{12}$$

Given a program with $t > b - c^e/\gamma$ that tolerates shirking, the fundamental threshold in equilibrium is

$$\theta^* = \frac{c - s}{(1-\gamma)(b - t)}. \tag{13}$$

In both cases, reducing the fundamental threshold to the first best $\theta^* = 0$ requires the subsidy s to be as close to c as possible. However, by the nature of the intervention program, this also requires the contingent tax $t = s$ to be as close to c as possible. The scale of the intervention program is constrained by the incentive constraint as shown in (10), and whether the constraint is binding depends on the severity of the moral hazard problem.

Assumption 2 *The moral hazard problem is severe,* $\frac{c^e}{\gamma} > b - c$.

Given Assumption 2 above, the maximum program size t that prevents shirking in the second period is strictly less than c, the cost of the investment project. Therefore, the government guarantee program cannot achieve efficient fundamental threshold in the first stage and prevent shirking in the second period at the same time. The result is summarized in Proposition 5 below. When Assumption 2 does not hold, the government guarantee program with $t = c$ achieves the first-best outcome.

Proposition 5 *Given Assumption 1 and 2, no government guarantee program can restore the first-best outcome when* $\sigma \to 0$*.*

Partial-participation Programs. Now let us consider a subsidy-tax program with $\frac{s}{t} \in [\frac{c}{b}, 1)$. Given that the tax is higher than the subsidy, whether to participate in the program depends on investors' idiosyncratic beliefs of the probability that the aggregate state is good. As in the benchmark model, the program is the most attractive to agents with intermediate beliefs. What complicates the analyses is that agents will take into account their effort decisions in the second period when they compare the cost and benefit of participating in the program. When the moral hazard problem in the second period is not severe, i.e., Assumption 2 does not hold, the policy maker can choose $s = c$ and $t \in [c, b)$ to implement the first-best outcome, which is the same as government guarantee programs. In the following analyses, we focus on the case when the moral hazard problem is severe, i.e., Assumption 2 holds, and full-participation government guarantee programs cannot achieve the first best.

Given that Assumption 2 holds and $t > c$, the optimistic agents will reject the intervention offer and exert effort, the agents with medium beliefs will accept the intervention offer and shirk. Intuitively, the intervention offer reduces participant's investment risk as well as "skin in the game". The most optimistic agents who strongly believe in the success of investment do not want to share the profits with the policy maker. Therefore, they will reject the offer and fully endogenize the payoff from investment which incentivizes them to make the first-best effort choice. In contrast, the agents with medium beliefs are willing to invest only if the policy maker bears part of the investment risk. However, the intervention program also reduces their "skin in the game" because they need to share the investment profits with the policy maker but bare the full cost of effort. As a result, these agents will participate in the intervention program and shirk.

Formally, given the optimal effort choices in the second stage, the expected payoffs from $\{a = 1, Reject\}$ and $\{a = 1, Accept\}$ for an agent who receives signal x_i and forms belief $\tilde{p}_i$ are

$$\mathbb{E}[\pi(\theta, l)|x_i] = \tilde{p}_i(b - c^e) - c, \tag{14}$$

$$\mathbb{E}[\tilde{\pi}(\theta, l)|x_i] = \tilde{p}_i(1 - \gamma)(b - t) - (c - s). \tag{15}$$

The expected payoffs are linear and increasing in the belief $\tilde{p}_i$, and the slopes are different. The difference in the slopes of $\mathbb{E}\pi(\theta, l)$ and $\mathbb{E}\tilde{\pi}(\theta, l)$,

$$(b - c^e) - (1 - \gamma)(b - t) = \gamma b + t(1 - \gamma) - c^e > 0 \tag{16}$$

is strictly positive given Assumption *1a*. Investing alone is the optimal choice if and only if the belief $\tilde{p}_i$ exceeds the critical participation belief

$$p_2^*(s, t) \equiv \frac{s}{\gamma b + t(1 - \gamma) - c^e}. \tag{17}$$

Not investing is the optimal action choice if and only if the belief of the agent is worse than the critical investment belief

$$p_1^*(s, t) \equiv \frac{c - s}{(1 - \gamma)(b - t)}. \tag{18}$$

The optimal action choice if the belief of success probability is between $p_1^*(s, t)$ and $p_2^*(s, t)$ is to invest and accept the offer.

Similar to those in the benchmark model, the critical beliefs determine the equilibrium thresholds of investment and participation regarding the private signal x. Investment efficiency in the first stage requires the critical investment belief $p_1^*(s, t)$ to be as close to 0 as possible, which implies that the policy maker should choose subsidy $s = c$. On the other hand, if t can be selected properly such that the critical participation belief $p_2^*(s, t) < 1$, the investors who are very optimistic about the aggregate state would choose to invest and reject the offer. The exclusion of optimistic investors from the program improves efficiency in the second stage game and reduces the policy maker's cost from inefficient failures due to shirking. As the information friction goes to zero, the mass of "pivotal" investors who participate in the program goes to zero. The following proposition summarizes the result.

Proposition 6 *Given Assumption 1 and 2, the equilibrium outcome given a subsidy-tax program (s, t) with $s = c$ and $\frac{c + c^e - \gamma b}{1 - \gamma} < t < b$ converges to the first best when $\sigma \to 0$. The ex-ante cost of providing such program also converges to 0 when $\sigma \to 0$.*

The above proposition demonstrates the advantage of the partial-participation programs compared with full-participation programs like government guarantee when the moral hazard problem is relatively severe. In the benchmark model, both types of programs can achieve the first-best outcome at zero cost with diminishing information friction if $\tau = 1$. They are different in terms of the program size: full-participation programs invite all investors, while partial-participation programs only target the "pivotal" investors. Absent other frictions, the size of a program does not alter the efficiency or the cost of implementing the program. However, the moral hazard problem causes welfare losses in proportion to the size of a program. When using a government guarantee program, the policy maker faces a trade-off between the first-stage investment efficiency and the second-stage effort efficiency. A program with high subsidy over tax ratio ($\frac{s}{t}$) encourages investment in the first stage but deters effort input in the second stage. This trade-off limits the role of the government guarantee program in improving social efficiency. On the contrary, despite the moral hazard problem, a partial-participation program still achieves the first-best outcome at zero cost. The advantage of partial-participation programs in dealing with moral hazard is that they only involve a small mass of investors. Although these participating investors shirk in the second stage, it will have a limited impact on the social welfare since the mass of these participating investors goes to zero as the information friction vanishes. In general, the partial-participation program proposed in this paper is superior to the full-participation programs such as government guarantee in the presence of any size-related inefficiency.

5 Extensions

5.1 Unobservable Ex-ante Heterogeneity

In this part, we study whether the existence of ex-ante heterogeneity in agents' payoff structure and information structure changes our results. The assumptions on the heterogeneity resemble those in Sakovics and Steiner (2012). Our analyses differ from their paper in two dimensions. First, they studied the optimal intervention when the policy maker can only provide a lump-sum subsidy, while we consider subsidy-tax programs. Second, they assume the types of agents are observable, while we allow for hidden types.

There are N groups of infinitesimal agents indexed by g, each group with mass m^g. There are three folds of heterogeneity. First, the agents differ in their profitability. They pay the same investment cost c yet earn different revenue b^g from successful investment. Assume there is no inefficient project, so $b^g > c$ for all g. Second, the agents impose different levels of externalities for the coordination results. Specifically, the aggregate action $l = \sum_{g=1}^{N} \int_0^{m^g} w^g a_i^g di$. Same as in the benchmark model, the condition that investment is successful is $l \geq 1 - \theta$. The weights are normalized such that $\sum_{g=1}^{N} w^g m^g = 1$. Lastly, each agent receives a private signal $x_i^g = \theta + \sigma\varepsilon_i^g$, where ε_i^g is independent across agents and follows a group-specific distribution with c.d.f. $F^g(\varepsilon)$, the support of which is $\left[-\frac{1}{2}, \frac{1}{2}\right]$. We assume an agent's group is not observable to the policy maker. However, the policy maker knows the composition of agents.

The equilibrium without intervention is summarized by the following proposition.

Proposition 7 *Without intervention, there is a unique equilibrium in which an agent in group g invests if and only if her private signal is greater or equal to* ξ_0^g*, which is given by*

$$\xi_0^g = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g} + \sigma F_g^{-1}\left(\frac{c}{b^g}\right). \tag{19}$$

From the above proposition, we can calculate the fundamental threshold θ^* above which the investments are successful. The expression for the fundamental threshold is given by

$$\theta^* = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g}, \tag{20}$$

which is a weighted average of the cost-benefit ratio of different types of agents. Let $b_{min} = \min \{b^g\}_{g=1}^N$. The following proposition shows our previous results still hold when there is unobservable heterogeneity among agents.

Proposition 8 *Given a subsidy-tax program with* $s < c$ *and* $s < t < b_{min}$*, there exists a unique equilibrium in which a type j agent follows the strategy below,*

$$\begin{aligned} a = 1, \textit{Reject}, \quad &\textit{if} \ \ x \geq \eta_g^*(s,t), \\ a = 1, \textit{Accept}, \quad &\textit{if} \ \ \xi_g^*(s,t) \leq x < \eta_g^*(s,t), \\ a = 0, \quad &\textit{if} \ \ x < \xi_g^*(s,t), \end{aligned}$$

where

$$\xi_g^*(s,t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t} + \sigma F_g^{-1}\left(\frac{c-s}{b_g - t}\right),$$
$$\eta_g^*(s,t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t} + \sigma F_g^{-1}\left(\frac{s}{t}\right).$$

When $s = c$ *and* $c < t < b_{min}$*, the equilibrium outcome converges to the first-best outcome and the expected cost of the program converges to 0 when* $\sigma \to 0$.

If agents also differ in the cost of investment, i.e., c^g can be different across groups, we need to relax the assumption that type are unobservable to the government. Instead, we assume the government can observe c_i for each individual agent. If $c_i = c^{g1} = c^{g2}$, the government does not need to know whether agent i is from $g1$ or $g2$. Under this setup, it is equivalent to solve the problem with $\tilde{c^g} = 1$ and $\tilde{b^g} = \frac{b^g}{c^g}$, then scale up agent i's offer by c_i^g.

The intuition for how our proposed intervention program works in the case with ex-ante heterogeneous agents is essentially the same as in the benchmark model. The intervention program incentivizes "pivotal" agents who originally choose not to invest to change their decisions. All agents knowing that there is an increase in the aggregate action l all believe in a higher probability of success. Amplified by higher-order beliefs, the intervention program can efficiently restore the first-best coordination results. Note that the notion of "pivotal" agents refers to the interim type of agents. Since different groups earn different profitabilities from successful investments, they require a different success probability to agree to invest. Our intervention program identifies and targets agents with beliefs right below the cutoffs of their own group. In Sakovics and Steiner (2012), they only look at direct subsidy programs and argue that an efficient program should target the ex-ante "pivotal" group, the group with low b^g and high w^g in our setup. Our results above demonstrate that by allowing an additional intervention tool, the contingent tax t, we are able to reduce coordination failure at a much lower cost. Moreover, the implementation of our proposed program does not require information on an agent's group, therefore our proposed program could save the potential cost of information acquisition.

5.2 General Payoff Structure

In this section, we follow the setups of the symmetric binary-action global games in Morris and Shin (2003) and allow for general monotonic payoff functions.

As in the benchmark model in section 2, an agent's payoff from not investing ($a_i = 0$) is normalized to zero. An agent's payoff from investing ($a_i = 1$) is modified to be a continuous function $\pi(x, l)$, which weakly increases in both the private signal x and the aggregate action $l = \int_0^1 a_i di$.[8] The fundamental θ follows a uniform distribution on $[\underline{\theta}, \bar{\theta}]$. The private signal received by agent i is $x_i = \theta + \sigma\varepsilon_i$, where ε_i are i.i.d. and has a density function $f(\varepsilon)$ and a distribution function $F(\varepsilon)$ with support $[-\frac{1}{2}, \frac{1}{2}]$.

For simplicity, we only consider the family of linear intervention programs. In general, we could allow transfer as a non-linear function of the agents' payoff. The intervention program (s, t) consists of two parts, a direct subsidy $s \geq 0$ and a proportional tax $t \in [0, 1]$. If an agent accepts the offer, she receives the direct upfront subsidy s and pays the proportional

[8]We assume the payoff is a function of the private signal instead of the fundamental for simplicity of demonstration. Our results still hold under the alternative setup. See Morris and Shin (2003) for the discussion of the two setups.

tax after the realization of the investment outcome. Her payoff from accepting the offer is[9]

$$\tilde{\pi}(x,l) = (1-t)\pi(x,l) + s. \tag{21}$$

Agents who receive low private signals believe in low realization of the fundamental θ and low aggregate action l, so they are pessimistic about their payoffs from investments. Therefore, they expect to pay low tax and are more willing to accept the offer than optimistic agents. Recall the partial-participation programs in the benchmark model. These programs do not appeal to the optimistic agents who do not need extra incentive to invest, which efficiently saves resources and reduces the cost of the program. The proportional tax t captures this feature and helps to target agents receiving medium signals.

We adopt the standard assumptions on the payoff function in the literature.

Assumption 3 *The payoff function $\pi(x,l)$ satisfy the following properties:*

1. *(Monotonicity) The payoff function $\pi(x,l)$ is weakly increasing in both arguments.*

2. *(Strict Laplacian State Monotonicity) $\int_0^1 \pi(x,l)\mathrm{d}l$ is strictly increasing in x.*

3. *(Limit Dominance) There exists $\theta_0, \theta_1 \in (\underline{\theta} + \frac{1}{2}\sigma, \bar{\theta} - \frac{1}{2}\sigma)$ such that*

$$\pi(x,1) < 0, \text{ for all } x < \theta_0, \tag{22}$$

$$\pi(x,0) > 0, \text{ for all } x > \theta_1, \tag{23}$$

4. *(Continuity) $\int_0^1 g(l)\pi(x,l)\mathrm{d}l$ is continuous in x for any density function g.*

The first assumption states the strategic complementarities among the investment choices of different agents. The individual payoff of investing increases when more agents invest. Also, a higher fundamental increases everyone's incentive to invest, given the same aggregate investment. Note that the payoff function need not be strictly increasing or continuous. For example, the payoff function in our benchmark model in Section 2 is a step function. The role of the second assumption is to make sure the equilibrium is unique when it exists, with or without the intervention program. The third assumption ensures the existence of two dominance regions so that we can adopt the iterated deletion of dominated strategies from both sides. The last assumption regulates integration of the payoff function so the equilibrium always exists.

[9] One might notice that when $\pi(x,l) < 0$, investors end up paying a negative "tax". In fact, let $\underline{\pi} = \pi(\underline{\theta} - \frac{1}{2}\sigma, 0)$ be the lower bound of the payoff. The intervention program can be implemented by providing a positive subsidy $s - t\underline{\pi}$ and imposing a proportional tax t on the positive tax base $\pi(x,l) - \underline{\pi}$.

The equilibrium without intervention is characterized in the proposition below. The "natural outcome" serves as a benchmark to analyze the effect of intervention programs.

Proposition 9 *Without intervention* ($s = t = 0$)*, when the information friction* σ *is small enough, there is a unique equilibrium in which each agent invests if and only if her private signal* $x \geq \xi_0^*$ *given by*

$$\int_0^1 \pi(\xi_0^*, l)dl = 0.$$

Compare the coordination results characterized in the above proposition with the first-best outcome. In the first-best scenario, if all agents investing can generate positive surplus, the social optimal outcome is for all agents to invest. In other words, the first-best scenario is that all agents follow the same cutoff strategy θ_0, the upper bound for the left dominance region. By Assumption 3, unless $\pi(\theta_0, l) = 0$ for any $l \in [0, 1]$, the natural coordination outcome $\xi^* > \theta_0$. Therefore if the realized fundamental $\theta \in (\xi^*, \theta_0)$, there would be a coordination failure. And the goal of intervention is to reduce the coordination threshold from ξ^* to as close to θ_0 as possible.

Next we analyze the equilibrium with an intervention program (s, t). We focus on the partial-participation programs and demonstrate its zero cost of implementation in the limiting case. Proposition 10 summarizes the conditions for such partial-participation programs.

Definition 1 *A intervention program* (s, t) *is a partial-participation program with target* ξ^* *if and only if it satisfy the following three conditions,*

1. *(Intervention Target)* $\int_0^1 \pi(\xi^*, l)dl = -\frac{s}{1-t}$.
2. *(Optimism Exclusion)* $\pi(\xi^*, 1) > \frac{s}{t}$,
3. *(Left Dominance Region)* $\pi(\underline{\theta}, 1) < -\frac{s}{1-t}$,

Denote a coordination game with information friction σ and intervention program (s, t) by $G(\sigma; s, t)$, we can prove the following proposition.

Proposition 10 *Given a partial-participation program* (s, t) *with target* ξ^**, the following two properties must be satisfied in any Bayesian Nash equilibrium of the coordination game* $G(\sigma; s, t)$*,*

1. *Agents invests if and only if their private signal* $x > \xi^*$*;*
2. *There exists a threshold* $\eta^*(\sigma)$ *such that investing agents strictly prefer not to participate in the intervention program if and only if their private signal* $x > \eta^*(\sigma)$*.*

When $\sigma \to 0$, $\eta^*(\sigma)$ converge to ξ^*.

The above proposition provides conditions under which there exist partial-participation programs to reduce the investment threshold to ξ^*. Same as in the benchmark regime-change model, in the limit, ex ante expected mass of participants goes to zero, which implies zero cost of implementation. The question remaining is whether there exist such programs to costlessly restore the first-best scenario, i.e. $\xi^* = \theta_0$. The proposition below answers this question.

Proposition 11 *If $\int_0^1 \pi(\theta_0, l)dl \geq \pi(\underline{\theta}, 1)$, for any $\xi^* \in (\theta_0, \xi_0^*)$, there exists a partial-participation program with target ξ^*.*

With the intervention program, all agents become more optimistic about their investment payoff. Therefore, the left dominance region, where agents prefer not to invest even if $l = 1$, shrinks. The condition specified in Proposition 11 guarantees that the left dominance region still exists with the intervention program. If the condition is violated, there might be multiple equilibria when targeting ξ^* close to θ_0. However, if we follow the equilibrium refinements proposed in Goldstein and Pauzner (2005), we can select the equilibrium described in Proposition 10 even without the left dominance region. Therefore, following the refinements, there always exists a partial-participation program that restores the first-best scenario. Moreover, the left dominance region may disappear because we limit our attention to programs with linear transfers. Linear transfer schedules generally gives a lot of subsidies in case of low fundamental. If the policy maker lowers subsidies in the case of very low fundamental realizations (when $\theta < \theta_0$) or adds convexity to the tax schedule properly, the left dominance region as well as the uniqueness of the equilibrium can be recovered. Either way, there always exists an intervention program that can restore the first-best scenario.

6 Selected Applications

The partial-participation programs can be applied to various contexts with coordination problems. In this section, we discuss three representative applications.

6.1 Debt Rollover

It has been widely recognized in the literature that panic-based debt run can leads to inefficient firm default. Specifically, consider a firm with many small debt-holders. The firm is more likely to survive if more debt-holders roll over their debts. Therefore, debt-holders' rollover decisions features strategic complementarities. When the fundamental of the firm is weak, debt-holders might stop rolling over their debts because they worry the others would also stop, which can leads to self-fulfilling debt run. Our analyses suggest tranching can be a cost-efficient way to reduce such coordination failure. Instead of one standard debt contract, the firm can issue two types of debts with different seniorities. The senior debt promises lower return yet provides higher payment than the junior debt when the firm defaults. Without tranching, debt-holders who have medium beliefs and coordination concerns would not roll over their standard debts. With the safer option of senior debt, they are willing to lend to the firm which eases the liquidity concern of the firm and boosts all debt-holders' beliefs in the firm's survival. This effect can be amplified by higher order beliefs. In equilibrium, only the pivotal debt-holders choose the senior option. However, the availability of the safer senior debt improves all debt-holders' belief in that the firm can raise enough funds to survive.

Bank run is another similar application. To implement the partial-participation programs, the government can offer optional but costly deposit insurance. In fact, Alipay, the largest online payment platform in China, offers all users an option to purchase insurance against losses on their associated financial accounts. The insurance is costly if their accounts are safe yet provides protection when the platform fails. Therefore, it would work in a similar way as the senior debt option to reduce coordination failure. It is less costly than the mandatory deposit insurance because it screens for the "pivotal depositors" and leaves out the optimistic depositors who would not run even without insurance protection.

6.2 Market Freeze

During the 2008 financial crisis, many financial institutions and investors significantly reduced their leverage. This process pushed down the market prices of Commercial Mortgage-Backed Securities (CMBS) and Residual Mortgage-Backed Securities (RMBS). The markets for RMBS and CMBS froze, and prices were well below their fundamentals. Among others, coordination failure can prevent the market from thawing. If only a few investors participate in the market for Mortgage-Backed Securities (MBS), the liquidity in the market is not enough to drive the prices back to the fundamental and the participating investors suffer losses on their investments. However, if a significant amount of liquidity is injected in the market, the prices are more likely to be driven back to reflect the fundamental and investors who bought at a discount can profit from the investment.

In March of 2009, the US Treasury announced the Legacy Securities Public-Private Investment Program (PPIP). Under the program, private equity was matched by government equity and debt to form Public-Private Investment Funds (PPIFs) and purchase highly rated legacy MBS from financial institutions. Private investors in the PPIFs effectively receive investment subsidies from the government and are levered up for their investment. They earn higher investment return in good times and are protected by limited liabilities in bad times. Hence, PPIP is uniformly beneficial to all qualifying private investors and can be interpreted as full-participation programs in our model. PPIP is not efficient in resource allocation in the sense that part of the government funding is provided to the optimistic investors who would have invested in MBS market without PPIP. According to our analyses, the government can reduce the cost of rejuvenating the market by offering a partial-participation program instead. Mapping into the context of PPIP, the government could offer to inject equity into PPIFs in proportion to debt holdings by private investors. This option of debt investment reduces the losses from freezing the MBS market. As a return, the government shares the profit of investment if the market for MBS is successfully rejuvenated. This offer incentivizes the pivotal investors to invest in the MBS market. Since all investors are aware of the offer, they know that the aggregate investment will increase and hence also have more incentive to invest.

6.3 Shopping Mall Investment

We analyze a real investment problem in this section. Pashigian and Gould (1998) documents the strategic complementarities among department stores in the same shopping mall. Specifically, department stores with reputations can bring in mall traffic and increase the sales of less-known stores. As discussed in Sakovics and Steiner (2012), the difference in reputation maps into w_g, the importance in coordination outcome of different groups in section 5.1.

Consider a newly opened shopping mall inviting different brands to open new stores. Since all stores benefit from customers' visit to the shopping mall, all stores' investment return increases in the occupancy ratio of the shopping mall. Therefore, coordination failure could lead to low occupancy ratio and failure of the shopping mall. In order to boost investment, according to our analyses, the shopping mall manager could offer an equity injection option. Specifically, if a brand accepts the equity injection offer and opens a new store in the shopping mall, the shopping mall manager pays part of the investment cost and receives proportional profit made by the store as a return. This offer is not appealing to the optimistic brands because they do not want to share the profits with the shopping mall. For brands that are around investment threshold, the equity injection offer reduces their investment risk and increases their expected payoff from the investment. Amplified by higher-order beliefs, all brands significantly lower their investment threshold. Moreover, in equilibrium, only the "pivotal" brands accept the offer. Therefore the resources to finance the intervention program are efficiently allocated.

It is reasonable to assume different brands have different profit functions. We have shown in section 5.2 that the interim critical agents who are around their own investment thresholds self-select to accept our offer. The result that the equity injection offer effectively reduces coordination failure and incurs low financing cost for the shopping mall owner still holds.

7 Conclusions

In this paper, we analyze a canonical coordination game under global games framework and propose a novel intervention program for a policy maker to reduce coordination failures. The intervention program screens for the marginal agents who receive medium signals, which reduces the cost of implementation for the program. At the same time, correctly incentivizing the marginal agents have a significant impact on all agents due to strategic complementarities and the amplification through higher order beliefs. In the limit of zero noise in agents' private signals, our proposed program eliminates all coordination failures at zero cost since the expected mass of marginal investors goes to zero. Compared with conventional government guarantee type of programs, our proposed program not only incurs lower cost of implementation but also is shown to be more robust to moral hazard problems.

We demonstrate with three examples that our proposed program has a wide range of applications in improving coordination failures. As a concluding remark, we would like to point out some limitations of the proposed program. First, the program requires the policy maker to observe and condition the provision of the program on agents' action choices, which might not be feasible. For example, in the context of panic-based currency attack, it is hard to trace the identities of the currency holders and give them an optional offer. Second, the effectiveness of the proposed program relies on agents' rationality. If agents possess bounded rationality, the amplification effect through higher order beliefs will be limited.

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Appendices

A Proofs

Proof of Proposition 1. It can be proved by iterated deletion of dominated strategies. Let $p(x;k)$ denote the interim belief of success when an agent receives private signal x and all other agents follow a cutoff investment strategy k as defined in (4). First, we want to show that strategies survive n rounds of iterated deletion of dominated strategies if and only if

$$a(x) = 0, \text{ if } x < \underline{\xi}_n, \tag{A.1}$$
$$\text{and } a(x) = 1, \text{ if } x \geq \bar{\xi}_n. \tag{A.2}$$

where $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ satisfies

$$-\infty = \underline{\xi}_0 < \underline{\xi}_1 \leq \cdots \leq \underline{\xi}_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \tag{A.3}$$

This result can be proved by induction. Let $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = +\infty$, so the first round of deletion starts with the full set of strategies. Suppose round $n \in \mathbb{N}$ of deletion has been completed. In round $n+1$, the best scenario for an agent to invest is that all other agents follow a cutoff strategy with threshold $\underline{\xi}_n$. Therefore, for any x such that $p(x;\underline{\xi}_n) < \frac{c}{b}$, $a(x) = 1$ is strictly worse than $a(x) = 0$. Similarly, the best scenario for an agent to choose $a_i = 1$ is that all other agents follow a cutoff strategy with threshold $\underline{\xi}_n$. As a result, for x such that $p(x;\bar{\xi}_n) > \frac{c}{b}$, any strategy profile with $a(x) = 1$ is strictly better than $a(x) = 0$.

Given $p(x;k)$ is non-decreasing in x, the strategy profiles that survives deletion of dominated strategies can be summarized in the form of (A.1)(A.2), with $(\underline{\xi}_{n+1}, \bar{\xi}_{n+1})$ defined inductively as

$$\underline{\xi}_{n+1} = \inf\left\{x : p(x;\underline{\xi}_n) \geq \frac{c}{b}\right\} \tag{A.4}$$

and

$$\bar{\xi}_{n+1} = \sup\left\{x : p(x;\bar{\xi}_n) \leq \frac{c}{b}\right\} \tag{A.5}$$

The monotonicity of $p(x;k)$ guarantees that $\underline{\xi}_{n+1} \leq \bar{\xi}_{n+1}$ given $\underline{\xi}_n \leq \bar{\xi}_n$. Note the dominance region assumption implies that $\underline{\xi}_1 > -\infty$ and $\bar{\xi}_1 < +\infty$ when σ is small enough. Therefore,

$\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ is a well-defined sequence of real couple which satisfies (A.3).

Now we've proved that $\{\underline{\xi}_n\}_{n=1}^{\infty}$ and $\{\bar{\xi}_n\}_{n=1}^{\infty}$ are both monotonic and bounded sequences. Thus, they converges to two finite numbers $\underline{\xi}$ and $\bar{\xi}$ respectively when $n \to \infty$. And the two limits satisfy

$$\underline{\xi} \leq \bar{\xi}. \tag{A.6}$$

The definition (A.4)(A.5) implies that $p(\underline{\xi}; \underline{\xi}) \geq \frac{c}{b}$ and $p(\bar{\xi}; \bar{\xi}) \leq \frac{c}{b}$. Note that

$$p(\xi; \xi) = F\left(\frac{\xi - \theta^*(\xi)}{\sigma}\right) = \theta^*(\xi), \tag{A.7}$$

is strictly increasing in ξ. Therefore $\underline{\xi} = \bar{\xi}$ must be the unique solution to $\theta^*(\xi) = \frac{c}{b}$, which is

$$\xi_0^* = \frac{c}{b} + \sigma F^{-1}\left(\frac{c}{b}\right). \tag{A.8}$$

Since there's only one strategy that survives the iterated deletion of dominated strategies, the equilibrium of the game is unique and the associated equilibrium strategy is the cutoff investment strategy with threshold ξ_0^*. ■

Lemma 1 *Suppose the optimal strategy of an agent as a function of her interim belief of success $\hat{p}_i$ can be characterized as*

$$\begin{aligned} a_i &= 1, \textit{Reject}, \quad \textit{if} \;\; \hat{p}_i > p_2^*, \\ a_i &= 1, \textit{Accept}, \quad \textit{if} \;\; p_1^* < \hat{p}_i \leq p_2^*, \\ a_i &= 0, \quad \textit{if} \;\; \hat{p}_i \leq p_1^*, \end{aligned}$$

where p_1^* and p_2^* are two threshold beliefs that satisfy $0 \leq p_1^* < p_2^* \leq 1$. There is a unique Bayesian Nash equilibrium and the equilibrium strategy of any agent is

$$\begin{aligned} a_i &= 1, \textit{Reject}, \quad \textit{if} \;\; x_i \geq \eta^*, \\ a_i &= 1, \textit{Accept}, \quad \textit{if} \;\; \xi^* \leq x_i < \eta^*, \\ a_i &= 0, \quad \textit{if} \;\; x_i < \xi^*, \end{aligned}$$

where $\xi^* = p_1^* + \sigma F^{-1}(p_1^*)$ and $\eta^* = p_1^* + \sigma F^{-1}(p_2^*)$

Proof of Lemma 1. We want to find a sequence $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ such that strategies survives

n rounds of iterated deletion of dominated strategies only if

$$a(x) = 0, \text{ if } x < \underline{\xi}_n, \tag{A.9}$$

$$\text{and } a(x) = 1, \text{ if } x \geq \bar{\xi}_n. \tag{A.10}$$

The reason that we can only iterate on the investment cutoff without keeping track of the participation decisions is that an agent's investment decision is independent of other agents' participation decisions. The recursive expression for $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ is

$$\underline{\xi}_{n+1} = \inf\{x : p(x; \underline{\xi}_n) \geq p_1^*\}, \tag{A.11}$$

$$\bar{\xi}_{n+1} = \sup\{x : p(x; \bar{\xi}_n) \leq p_1^*\}. \tag{A.12}$$

Applying the same techniques in the proof of Proposition 1, it becomes clear that the limit of the two cutoff sequences converges to

$$\xi^*(s,t) = p_1^* + \sigma F^{-1}\left(p_1^*\right), \tag{A.13}$$

which is the investment cutoff in the unique Bayesian Nash equilibrium of the global game. The associated participation cutoff η is the solution to

$$p(\eta; \xi^*(s,t)) = p_2^*. \tag{A.14}$$

Solving the above equation yields

$$\eta^*(s,t) = p_1^* + \sigma F^{-1}\left(p_2^*\right). \tag{A.15}$$

■

Proof of Proposition 2. In case 1, *invest-and-reject* is dominated by *invest-and-accept*. Therefore, we can rewrite the investment payoff by letting $b' = b - t$ and $c' = c - s$ and directly apply Proposition 1. Similarly, *invest-and-accept* is jointly dominated by *invest-and-accept* and *not-invest* in case 3. Since the intervention program is never going to be accepted, the equilibrium is the same as that described in Proposition 1. Case 2 is a direct implication of Lemma 1. ■

Proof of Proposition 3.

As specified in equation 5, with program (s,t), the fundamental cutoff is $\frac{c-s}{b-t}$. Therefore, the programs targeting at the first-best fundamental cutoff 0 should satisfy $s = c$. Hence, the subsidy to tax ratio of a program targeting at the first best is $\frac{s}{t} = \frac{c}{t}$. If the ratio is greater than 1, the program is a full-participation program. Otherwise, it is a partial-participation program.

As a result, if (s, t) satisfies the following two conditions, it is a full-participation program targeting the first best.

1. $0 \leq t \leq c$,

2. $s = c$.

If (s', t') satisfies the following two conditions, it is a partial-participation program targeting the first best.

1. $c < t' \leq b$,

2. $s' = c$.

Lastly, we calculate the limit of the cost functions as specified in equation 7 and 8. For any $\theta > 0$,

$$\lim_{\sigma \to 0} C(\theta, s, t) = \lim_{\sigma \to 0} (\tau s - t) \left[1 - F\left(\frac{0 - \theta}{\sigma} + F^{-1}(0) \right) \right] = (\tau s - t) \left[1 - F(-\infty) \right] = \tau s - t$$

$$\begin{aligned} \lim_{\sigma \to 0} C(\theta, s', t') &= \lim_{\sigma \to 0} (\tau s' - t') \left[F\left(\frac{0 - \theta}{\sigma} + F^{-1}\left(\frac{s'}{t'} \right) \right) - F\left(\frac{0 - \theta}{\sigma} + F^{-1}(0) \right) \right] \\ &= (\tau s' - t') \left[F(-\infty) - F(-\infty) \right] = 0 \end{aligned}$$

If $\theta = 0$,

$$\lim_{\sigma \to 0} C(\theta, s, t) = \lim_{\sigma \to 0} (\tau s - t) \left[1 - F\left(F^{-1}(0) \right) \right] = \tau s - t$$

$$\lim_{\sigma \to 0} C(\theta, s', t') = \lim_{\sigma \to 0} (\tau s' - t') \left[F\left(F^{-1}\left(\frac{s'}{t'} \right) \right) - F\left(F^{-1}(0) \right) \right] = \frac{s'}{t'} (\tau s' - t')$$

The cost of a partial-participation program is strictly less than that of a full-participation program.

$$\frac{s'}{t'} (\tau s' - t') = \tau c \frac{c}{t'} - c < \tau c - c \leq \tau s - t$$

For any $\theta < 0$,

$$\lim_{\sigma \to 0} C(\theta, s, t) = \lim_{\sigma \to 0} \tau s \left[1 - F\left(\frac{0 - \theta}{\sigma} + F^{-1}(0) \right) \right] = \tau s \left[1 - F(\infty) \right] = 0$$

$$\lim_{\sigma\to 0} C(\theta, s', t') = \lim_{\sigma\to 0} \tau s' \left[F\left(\frac{0-\theta}{\sigma} + F^{-1}\left(\frac{s'}{t'}\right)\right) - F\left(\frac{0-\theta}{\sigma} + F^{-1}(0)\right)\right]$$
$$= \tau s' \left[F(\infty) - F(\infty)\right] = 0$$

■

Proof of Proposition 4. We compare the expected cost of a full-participation program with (s,t) a partial-participation program (s',t') with small enough $\lambda > 0$.

The expected cost of the full-participation program is

$$\mathbb{E}_\theta[C(\theta,s,t)] = \frac{\tau s}{\bar\theta - \underline\theta}\int_{\underline\theta}^{\bar\theta}\left[1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right]\mathrm{d}\theta - \frac{t}{\bar\theta - \underline\theta}\int_{\theta^*}^{\bar\theta}\left[1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right]\mathrm{d}\theta,$$

and that of the partial-participation program (s',t'),

$$\mathbb{E}_\theta[C(\theta,s',t')] = \frac{\tau s'}{\bar\theta - \underline\theta}\int_{\underline\theta}^{\bar\theta}\left[F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right]\mathrm{d}\theta - \frac{t'}{\bar\theta - \underline\theta}\int_{\theta^*}^{\bar\theta}\left[F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right]\mathrm{d}\theta,$$

where ξ^* and η^* are the investment threshold and participation threshold defined as in Proposition 2, $\xi^* = \theta^* + \sigma F^{-1}(\theta^*)$, $\eta^*(s',t') = \theta^* + \sigma F^{-1}\left(\frac{s'}{t'}\right)$. To suppress notations, we omit the dependence of η^* on (s',t'). The difference between the cost of full-participation program (s,t) and that of partial-participation program (s',t') can be decomposed into two parts, $\mathbb{E}_\theta[C(\theta,s,t)] - \mathbb{E}_\theta[C(\theta,s',t')] = \Delta_1 + \Delta_2$, where

$$\Delta_1 = \frac{\tau s - t}{\bar\theta - \underline\theta}\int_{\theta^*}^{\bar\theta}\left[1 - F\left(\frac{\eta^* - \theta}{\sigma}\right)\right]\mathrm{d}\theta + \frac{\tau s}{\bar\theta - \underline\theta}\int_{\underline\theta}^{\theta^*}\left[1 - F\left(\frac{\eta^* - \theta}{\sigma}\right)\right]\mathrm{d}\theta,$$
$$\Delta_2 = -\frac{\tau\theta^*(t'-t)}{\bar\theta - \underline\theta}\int_{\underline\theta}^{\bar\theta}\left[F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right]\mathrm{d}\theta + \frac{t'-t}{\bar\theta - \underline\theta}\int_{\theta^*}^{\bar\theta}\left[F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right]\mathrm{d}\theta.$$

Δ_1 and Δ_2 are the cost difference on the extensive margin and intensive margin respectively.

Notice $\mathbb{E}[C(\theta,s,t)]$ is linear in s and t. Therefore, the expected cost of any full-participation program lies between the cost of the guarantee program $\lambda_1 = 0$ with $(s,t) = (\frac{c-\theta^* b}{1-\theta^*}, \frac{c-\theta^* b}{1-\theta^*})$, and the pure subsidy program $\lambda_2 = -\frac{c-\theta^* b}{1-\theta^*}$, with $(s,t) = (c - \theta^* b, 0)$. In the remaining part of the proof, we show that if either of the two conditions is satisfied, the proposed partial-participation program $(s',t') = (\frac{c-\theta^* b}{1-\theta^*} + \theta^*\lambda, \frac{c-\theta^* b}{1-\theta^*} + \lambda)$ with small positive λ has lower cost than both the guarantee program and the pure subsidy program.

Consider the pure subsidy program $(s,t) = (c - \theta^* b, 0)$. Plugging (s,t) into the expression of Δ_1, we have

$$\begin{aligned}
\Delta_1 =& \frac{\tau(c-\theta^* b)}{\bar{\theta}-\underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left[1 - F\left(\frac{\eta^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta, \\
=& \frac{\tau(c-\theta^* b)}{\bar{\theta}-\underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \int_{\eta^*}^{\bar{\theta}+\frac{1}{2}\sigma} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \mathrm{d}x\mathrm{d}\theta, \\
=& \frac{\tau(c-\theta^* b)}{\bar{\theta}-\underline{\theta}} \int_{\eta^*}^{\bar{\theta}+\frac{1}{2}\sigma} \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \mathrm{d}\theta\mathrm{d}x, \\
=& \frac{\tau(c-\theta^* b)}{\bar{\theta}-\underline{\theta}} \int_{\eta^*}^{\bar{\theta}+\frac{1}{2}\sigma} \left[1 - F\left(\frac{x-\bar{\theta}}{\sigma}\right)\right] \mathrm{d}x, \\
=& \frac{\tau(c-\theta^* b)}{\bar{\theta}-\underline{\theta}} \left[\bar{\theta} + \frac{1}{2}\sigma - \eta^* - \sigma \int_{-\frac{1}{2}}^{\frac{1}{2}} F(y)\mathrm{d}y\right] > \frac{\tau(c-\theta^* b)}{\bar{\theta}-\underline{\theta}}(1-\theta^*),
\end{aligned}$$

which is strictly positive.

For Δ_2, notice

$$\begin{aligned}
\int_{\alpha}^{\beta} \left[F\left(\frac{\eta^*-\theta}{\sigma}\right) - F\left(\frac{\xi^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta =& \int_{\alpha}^{\beta} \int_{\xi^*}^{\eta^*} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \mathrm{d}x\mathrm{d}\theta, \\
=& \int_{\xi^*}^{\eta^*} \int_{\alpha}^{\beta} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \mathrm{d}\theta\mathrm{d}x, \\
=& \int_{\xi^*}^{\eta^*} \left[F\left(\frac{x-\alpha}{\sigma}\right) - F\left(\frac{x-\beta}{\sigma}\right)\right] \mathrm{d}x,
\end{aligned}$$

therefore

$$\begin{aligned}
\Delta_2 =& \left(\frac{c-\theta^* b}{1-\theta^*} + \varepsilon\right) \frac{1}{\bar{\theta}-\underline{\theta}} \left[\int_{\xi^*}^{\eta^*} F\left(\frac{x-\theta^*}{\sigma}\right) \mathrm{d}x - \tau\theta^*(\eta^*-\xi^*)\right], \\
=& \left(\frac{c-\theta^* b}{1-\theta^*} + \varepsilon\right) \frac{\theta^*(\eta^*-\xi^*)}{\bar{\theta}-\underline{\theta}} \left[\int_{F^{-1}(\theta^*)}^{F^{-1}\left(\frac{s'}{t'}\right)} \frac{F(y)}{\theta^*(F^{-1}\left(\frac{s'}{t'}\right) - F^{-1}(\theta^*))} \mathrm{d}y - \tau\right], \\
=& \left(\frac{c-\theta^* b}{1-\theta^*} + \varepsilon\right) \frac{\theta^*(\eta^*-\xi^*)}{\bar{\theta}-\underline{\theta}} \left[G\left(\theta^*, \frac{s'}{t'}\right) - \tau\right].
\end{aligned}$$

Taking λ to 0, we have

$$\lim_{\lambda\to 0^+} \Delta_2 = \left(\frac{c-\theta^* b}{1-\theta^*}\right) \frac{\theta^*\sigma(\frac{1}{2} - F^{-1}(\theta^*))}{\bar{\theta}-\underline{\theta}} [G(\theta^*, 1) - \tau] = \frac{c-\theta^* b}{\bar{\theta}-\underline{\theta}} \theta^*\sigma[G(\theta^*, 1) - \tau].$$

If the first condition holds, $\tau < G(\theta^*, 1)$, $\lim_{\varepsilon\to 0^+} \Delta_2 > 0$, $\Delta_1 + \Delta_2$ is strictly positive for small enough λ. Also, if the second condition holds, $\theta^* + \sigma < 1$,

$$\lim_{\lambda \to 0^+} \Delta_1 + \Delta_2 > \frac{\tau(c - \theta^* b)}{\bar{\theta} - \underline{\theta}}(1 - \theta^* - \theta^* \sigma) > 0.$$

Now let's turn to the guarantee program with $s = t = \frac{c-\theta^* b}{1-\theta^*}$. For Δ_1, since $\eta^* = \theta^* + \sigma F^{-1}(\frac{s'}{t'}) < \theta^* + \frac{1}{2}\sigma$, we have

$$\Delta_1 > \frac{(\tau-1)s}{\bar{\theta} - \underline{\theta}} \int_{\theta^*}^{\bar{\theta}} \left[1 - F\left(\frac{\theta^* + \frac{1}{2}\sigma - \theta}{\sigma}\right)\right] \mathrm{d}\theta + \frac{\tau s}{\bar{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\theta^*} \left[1 - F\left(\frac{\theta^* + \frac{1}{2}\sigma - \theta}{\sigma}\right)\right] \mathrm{d}\theta \geq 0.$$

The last inequality is strict when $\tau > 1$. For Δ_2, we have

$$\Delta_2 = \lambda \frac{\sigma \theta^* (F^{-1}\left(\frac{s'}{t'}\right) - F^{-1}(\theta^*))}{\bar{\theta} - \underline{\theta}} \left[G\left(\theta^*, \frac{s'}{t'}\right) - \tau\right].$$

If $\tau > 1$, $\lim_{\lambda \to 0^+} \Delta_1 > 0$, $\lim_{\lambda \to 0^+} \Delta_2 = 0$. Thus, $\mathbb{E}_\theta[C(\theta, s, t)] - \mathbb{E}_\theta[C(\theta, s', t')] = \Delta_1 + \Delta_2 > 0$ for small enough λ.

If $\tau = 1$, since $\frac{s'}{t'} > \frac{c}{b} > \theta^*$, $G\left(\theta^*, \frac{s'}{t'}\right) > 1 = \tau$, $\Delta_2 > 0$ for any positive λ. Combining with $\Delta_1 \geq 0$, we have $\mathbb{E}_\theta[C(\theta, s, t)] - \mathbb{E}_\theta[C(\theta, s', t')] = \Delta_1 + \Delta_2 > 0$ for any positve λ.

To sum up, in either case, when λ being positve and small enough, the partial particiaption program $(s', t') = (\frac{c-\theta^* b}{1-\theta^*} + \theta^* \lambda, \frac{c-\theta^* b}{1-\theta^*} + \lambda)$ has lower expected cost than any full-participation program targeting θ^*. ■

Proof of Proposition 6. If we can choose (s, t) properly such that $0 < p_1^*(s, t) < p_2^*(s, t) < 1$, Lemma 1 implies in the unique Bayesian Nash equilibrium, agents follow a threshold strategy

$$\begin{aligned} a_i &= 1, \text{Reject}, \text{ if } x_i \geq \eta^*(s, t), \\ a_i &= 1, \text{Accept}, \text{ if } \xi^*(s, t) \leq x_i < \eta^*(s, t), \\ a_i &= 0, \text{ if } x_i < \xi^*(s, t), \end{aligned}$$

where

$$\begin{aligned} \xi^*(s, t) &= p_1^*(s, t) + \sigma F^{-1}(p_1^*(s, t)), \\ \eta^*(s, t) &= p_1^*(s, t) + \sigma F^{-1}(p_2^*(s, t)). \end{aligned}$$

Moreover, $\xi^*(s, t)$ and $\eta^*(s, t)$ both converges to $p_1^*(s, t)$ when $\sigma \to 0$. Thus, for any continuous belief of the fundamental held by the government, the ex-ante cost of the program converges to 0 when $\sigma \to 0$.

Now we want to show that it is possible to choose (s, t) such that $0 < p_1^*(s,t) < p_2^*(s,t) < 1$ and $p_1^*(s,t)$ can be arbitrarily close to 0. Let $s = c - \varepsilon$ and $\frac{c+c^e-\gamma b}{1-\gamma} < t < b$. The choice of t is feasible since Assumption 1b implies $\frac{c+c^e-\gamma b}{1-\gamma} < b$. Note $\frac{c+c^e-\gamma b}{1-\gamma} < t$ implies

$$
\begin{aligned}
p_2^*(s,t) &= \frac{s}{\gamma b + t(1-\gamma) - c^e} < \frac{c-\varepsilon}{c}, \\
p_1^*(s,t) &= \frac{c-s}{(1-\gamma)(b-t)} = \frac{\varepsilon}{(1-\gamma)(b-t)}.
\end{aligned}
$$

Therefore, for any fixed t, when $\varepsilon \to 0$, $p_1^*(s,t)$ converges to 0 and $p_2^*(s,t)$ converges to a positive number which is strictly less than 1. ■

Proof of Proposition 7. The proof is similar to the proof of Lemma 1. We want to find a sequence $\left\{(\underline{\xi}_n^g, \bar{\xi}_n^g)_{g=1}^N\right\}_{n=0}^{\infty}$ such that the strategies of group g agents survive n rounds of iterated deletion of dominated strategies only if

$$a^g(x) = 0, \text{ if } x < \underline{\xi}_n^g, \tag{A.16}$$

$$\text{and } a^g(x) = 1, \text{ if } x \geq \bar{\xi}_n^g. \tag{A.17}$$

To simplify notations, let $\underline{\xi}_n = (\underline{\xi}_n^g)_{g=1}^N$ and $\bar{\xi}_n = (\bar{\xi}_n^g)_{g=1}^N$ be the vectors of threshold signals. The recursive expression for $\left\{(\underline{\xi}_n^g, \bar{\xi}_n^g)_{g=1}^N\right\}_{n=0}^{\infty}$ is

$$\underline{\xi}_{n+1}^g = \inf_x \{x : p^g(x; \underline{\xi}_n) \geq \frac{c}{b^g}\}, \tag{A.18}$$

$$\bar{\xi}_{n+1}^g = \sup_x \{x : p^g(x; \bar{\xi}_n) \leq \frac{c}{b^g}\}. \tag{A.19}$$

We can prove by induction that

$$-\infty = \underline{\xi}_0 < \underline{\xi}_1 \leq \cdots \leq \underline{\xi}_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \tag{A.20}$$

Since any bounded monotonic sequence has a finite limit, take n to ∞, we have

$$\bar{\xi} \geq \underline{\xi}. \tag{A.21}$$

Now we want to show $\bar{\xi} = \underline{\xi}$. It can be proved by contradiction. Suppose $\bar{\xi} > \underline{\xi}$. Let h

be the group such that $\bar{\xi}^h - \underline{\xi}^h = \max_g \left\{\bar{\xi}^g - \underline{\xi}^g\right\} > 0$. Note that $\theta^*(\bar{\xi})$ is the solution to

$$\sum_{g=1}^{N} w^g m^g F^g \left(\frac{\bar{\xi}^g - \theta}{\sigma}\right) = \theta. \tag{A.22}$$

Therefore, $\theta^*(\bar{\xi}) - (\bar{\xi}^h - \underline{\xi}^h)$ is the solution to

$$\sum_{g=1}^{N} w^g m^g F^g \left(\frac{\bar{\xi}^g - (\bar{\xi}^h - \underline{\xi}^h) - \theta}{\sigma}\right) - \theta - (\bar{\xi}^h - \underline{\xi}^h) = 0. \tag{A.23}$$

Also notice $\theta^*(\underline{\xi})$ is the solution to

$$\sum_{g=1}^{N} w^g m^g F^g \left(\frac{\underline{\xi}^g - \theta}{\sigma}\right) - \theta = 0. \tag{A.24}$$

Let's compare (A.23) and (A.24). Since $\underline{\xi}^g > \bar{\xi}^g - (\bar{\xi}^h - \underline{\xi}^h)$ and $\bar{\xi}^h - \underline{\xi}^h > 0$, the left hand side of (A.24) is strictly larger than the left hand side of (A.23) for any given θ. Given the left hand side of (A.24) is strictly decreasing in θ, we must have $\theta^*(\bar{\xi}) - (\bar{\xi}^h - \underline{\xi}^h) < \theta^*(\underline{\xi})$. Therefore,

$$\begin{aligned}
p^h(\bar{\xi}^h; \bar{\xi}) =& Pr^h[\theta > \theta^*(\bar{\xi}) | \bar{\xi}^h], \\
=& F^h \left(\frac{\bar{\xi}^h - \theta^*(\bar{\xi})}{\sigma}\right), \\
=& F^h \left(\frac{\underline{\xi}^h - [\theta^*(\bar{\xi}) - (\bar{\xi}^h - \underline{\xi}^h)]}{\sigma}\right), \\
>& F^h \left(\frac{\underline{\xi}^h - \theta^*(\theta^*(\underline{\xi}))}{\sigma}\right), \\
=& p^h(\underline{\xi}^h; \underline{\xi}).
\end{aligned}$$

However. (A.18) and (A.19) implies $p^h(\bar{\xi}^h; \bar{\xi}) = p^h(\underline{\xi}^h; \underline{\xi}) = \frac{c}{b^h}$. Contradiction. This implies $\bar{\xi} = \underline{\xi} = \xi_0$.

To solve for ξ_0, note ξ_0 and θ_0 are the solutions to

$$\sum_{g=1}^{N} w^g m^g F^g \left(\frac{\xi^g - \theta}{\sigma}\right) = \theta, \tag{A.25}$$

$$F^g \left(\frac{\xi^g - \theta}{\sigma}\right) = \frac{c}{b^g}, \quad \text{for any } g = 1, \ldots, N. \tag{A.26}$$

Plugging (A.26) into (A.25) we have

$$\theta_0 = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g}, \tag{A.27}$$

$$\xi_0^g = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g} + \sigma F_g^{-1}\left(\frac{c}{b^g}\right), \quad \text{for any } g = 1, \dots, N. \tag{A.28}$$

■

Proof of Proposition 8. The optimal response of an agent in group g is

$$\begin{aligned} a_i = 1, \text{Reject}, &\quad \text{if } \hat{p}_i \geq \frac{s}{t}, \\ a_i = 1, \text{Accept}, &\quad \text{if } \frac{c-s}{b^g - t} \leq \hat{p}_i < \frac{s}{t}, \\ a_i = 0, &\quad \text{if } \hat{p}_i < \frac{c-s}{b^g - t}; \end{aligned}$$

We can apply the same method in the proof of Proposition 7 and show that in any equilibrium, agents of group g invest if and only if their private signal is greater or equal to

$$\xi_g^*(s,t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t} + \sigma F_g^{-1}\left(\frac{c-s}{b_g - t}\right). \tag{A.29}$$

Given the investment thresholds, we know the fundamental threshold above which there will be successful investment is

$$\theta^*(s,t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t}. \tag{A.30}$$

Therefore, the signal $\eta^*(s,t)$ that makes an agent from group g indifferent between accepting and rejecting the intervention program is

$$\eta^*(s,t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t} + \sigma F_g^{-1}\left(\frac{s}{t}\right). \tag{A.31}$$

■

Proof of Proposition 9. Consider an agent who receives private signal x and knows that all other agents invest if and only if observing private signal k. The expected payoff from investing is

$$U(k,x) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \pi\left(\theta, 1 - F\left(\frac{k-\theta}{\sigma}\right)\right) d\theta$$

Note that $U(k,x)$ weakly decreases in k and weakly increases in x. Intuitively, an agent has higher expected payoff if everyone else is more willing to invest or the agent receives a high signal indicating a high fundamental θ. Also note that $U(-\infty, x) < 0$ for $x < \theta_0$ and $U(+\infty, x) > 0$ for $x > \theta_1$.

Next we prove the uniqueness of equilibrium by iterated deletion of dominated strategies. The strategy profile of an agent is the action as a function of the private signal received. We denote it by $a(x) : \mathbb{R} \to \{0, 1\}$. We will prove that strategy survives n rounds of iterated deletion of dominated strategies if and only if

$$a(x) = 0, \text{ if } x < \underline{\xi}_n, \tag{A.32}$$

$$\text{and } a(x) = 1, \text{ if } x \geq \bar{\xi}_n. \tag{A.33}$$

where $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ satisfies

$$-\infty = \underline{\xi}_0 < \underline{\xi}_1 \leq \cdots \leq \underline{\xi}_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \tag{A.34}$$

This result can be proved by induction. Let the starting node be $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = +\infty$, meaning that there is no restrictions on agents' strategy. Suppose round $n \in \mathbb{N}$ of deletion has been completed. In round $n+1$, the most optimistic belief for an agent is that all other agents follow a cutoff strategy with threshold $\underline{\xi}_n$. Therefore, for any x such that $U(\underline{\xi}_n, x) < 0$, $a(x) = 1$ is strictly dominated by $a(x) = 0$. Similarly, the most pessimistic belief for an agent is that all other agents follow a cutoff strategy with threshold $\bar{\xi}_n$. As a result, for x such that $U(\bar{\xi}_n, x) > 0$, any strategy profile with $a(x) = 0$ is strictly dominated by $a(x) = 1$.

Given $U(k,x)$ is non-decreasing in x, the strategy profiles that survives deletion of dominated strategies must satisfy the restrictions in (A.32) and (A.33), with $(\underline{\xi}_{n+1}, \bar{\xi}_{n+1})$ defined inductively as

$$\underline{\xi}_{n+1} = \inf\{x : U(\underline{\xi}_n, x) \geq 0\} \tag{A.35}$$

and

$$\bar{\xi}_{n+1} = \sup\{x : U(\bar{\xi}_n, x) \leq 0\} \tag{A.36}$$

The monotonicity of $U(k,x)$ guarantees that $\underline{\xi}_{n+1} \leq \bar{\xi}_{n+1}$. Note that the dominance region assumption implies that $\underline{\xi}_1 > -\infty$ and $\bar{\xi}_1 < +\infty$. Therefore, $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ is a well-defined sequence of real couples which satisfies (A.34).

Now we've proved that $\{\underline{\xi}_n\}_{n=1}^{\infty}$ and $\{\bar{\xi}_n\}_{n=1}^{\infty}$ are both monotonic and bounded sequences. Thus, they converges to two finite numbers $\underline{\xi}$ and $\bar{\xi}$ respectively when $n \to \infty$. The definition (A.35) and (A.36) imply that $U(\underline{\xi}, \underline{\xi}) \geq 0$ and $U(\bar{\xi}, \bar{\xi}) \leq 0$. Notice for $y \in [\theta_0, \theta_1]$,

$$U(y, y) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{y-\theta}{\sigma}\right) \pi\left(y, 1 - F\left(\frac{y-\theta}{\sigma}\right)\right) d\theta = \int_0^1 \pi\left(y, l\right) dl,$$

strictly increases in y and $\underline{\xi} \leq \bar{\xi}$, it must be the case that $U(\underline{\xi}, \underline{\xi}) = U(\bar{\xi}, \bar{\xi}) = 0$.

Since $U(y, y)$ is continuous in y, $U(\underline{\theta}, \underline{\theta}) \leq 0$, $U(\bar{\theta}, \bar{\theta}) \geq 0$, there is a unique solution to $U(y, y) = \int_0^1 \pi\left(y, l\right) dl = 0$. Denote the solution by ξ_0^*, and we have $\underline{\xi} = \bar{\xi} = \xi_0^*$. Therefore, the only strategy that survives the iterated deletion of dominated strategies is the cutoff investment strategy with cutoff ξ_0^*. ■

Proof of Proposition 10. Consider an agent who receives private signal x and knows that all other agents invest if and only if their signal is above k. The expected payoff from investing and rejecting the intervention offer is

$$U^R(k, x) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \pi\left(\theta, 1 - F\left(\frac{k-\theta}{\sigma}\right)\right) d\theta.$$

The expected payoff from investing and accepting the offer is

$$U^A(k, x) = (1-t)U^R(k, x) + s$$

Therefore, the maximum expected payoff from investing is

$$U(k, x) = \max\{U^R(k, x), U^A(k, x)\}$$

We prove a lemma that will be useful later.

Lemma 2 *Given that all other agents invest if and only if their signal is above k, there exist two functions $k_1^*(k)$ and $k_2^*(k)$ such that an agent strictly prefers not investing if her private signal $x < k_1^*(k)$ and strictly prefers investing if $x > k_2^*(k)$. $k_1^*(k)$ and $k_2^*(k)$ are given by*

$$k_1^*(k) = \inf\left\{k^* : U^R(k, k^*) \geq -\frac{s}{1-t}\right\},$$

$$k_2^*(k) = \sup\left\{k^* : U^R(k, k^*) \leq -\frac{s}{1-t}\right\},$$

Both $k_1^*(k)$ and $k_2^*(k)$ are weakly increasing in k.

Proof of Lemma 2. The Left Dominance Region assumption in Definition 1 and Limit Dominance in Assumption 3 make sure that the two function $k_1^*(k)$ and $k_2^*(k)$ are well defined. By continuity of $U^R(k,x)$ in x, we have

$$U^R(k, k_1^*(k)) = U^R(k, k_2^*(k)) = -\frac{s}{1-t},$$

For any $x < k_1^*(k)$, $U^R(k,x) < -\frac{s}{1-t}$, $U^A(k,x) = (1-t)U^R(k,x) + s < 0$. Therefore, $U(k,x) = \max\{U^R(k,x), U^A(k,x)\} < 0$, the agent will not invest if observing $x < k_1^*(k)$. On the other hand, for any $x > k_2^*(k)$, $U^R(k,x) > -\frac{s}{1-t}$, $U^A(k,x) = (1-t)U^R(k,x) + s > 0$. Therefore, $U(k,x) = \max\{U^R(k,x), U^A(k,x)\} > 0$, the agent will invest after observing signal $x > k_2^*(k)$.

Since $U^R(k,x)$ is weakly decreasing in k, we can easily show that both $k_1^*(k)$ and $k_2^*(k)$ are weakly increasing in k. ■

With Lemma 2, we can prove the uniqueness of equilibrium by iterated deletion of dominated strategies. Denote the investment strategy by $a(x)$. We want to show a strategy survives n rounds of iterated deletion of dominated strategies if and only if

$$a(x) = \begin{cases} 0, & \text{if } x < \underline{\xi}_n, \\ 1, & \text{if } x > \bar{\xi}_n, \end{cases}$$

where $\underline{\xi}_0 = -\infty$, $\bar{\xi}_0 = \infty$. $\underline{\xi}_n$ and $\bar{\xi}_n$ are defined inductively by $\underline{\xi}_{n+1} = k_1^*(\underline{\xi}_n)$, $\bar{\xi}_{n+1} = k_2^*(\bar{\xi}_n)$.

Since $k^*(\xi)$ increases in ξ, $\underline{\xi}_n$ and $\bar{\xi}_n$ are increasing and decreasing sequences, respectively. As $n \to \infty$, $\underline{\xi}_n \to \underline{\xi}$ and $\bar{\xi}_n \to \bar{\xi}$. Therefore, $\underline{\xi} = k_1^*(\underline{\xi})$ and $\bar{\xi} = k_2^*(\bar{\xi})$. $\underline{\xi}$ and $\bar{\xi}$ must both be the solution to

$$U^R(\xi, \xi) = -\frac{s}{1-t}.$$

Let $l = 1 - F\left(\frac{\xi - \theta}{\sigma}\right)$, the equation can be written as

$$\int_0^1 \pi\left(\xi, l\right) dl = -\frac{s}{1-t} \tag{A.37}$$

By Strict Laplacian State Monotonicity in Assumption 3, the left hand side is continuous and strictly increasing in ξ. Also, $\int_0^1 \pi\left(\underline{\theta}, l\right) dl < -\frac{s}{1-t}$, $\int_0^1 \pi\left(\bar{\theta}, l\right) dl > 0 \geq -\frac{s}{1-t}$, there is a unique solution to the equation above, $\underline{\xi} = \bar{\xi} = \xi^*$. Notice ξ^* is independent of σ. Then by iterated deletion of dominated strategies, it is the unique investment cutoff in equilibrium.

Given the investment cutoff, we can solve for the private signal x such that $U^A(\xi^*, x) = U^R(\xi^*, x)$, or equivalently $U^R(\xi^*, \eta^*(\sigma)) = \frac{s}{t}$. Let $\eta^*(\sigma)$ be the maximum value that satisfies

$$U^R(\xi^*, \eta^*) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\eta^* - \theta}{\sigma}\right) \pi\left(\eta^*, 1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right) d\theta = \frac{s}{t}. \tag{A.38}$$

For any signal $x > \eta^*(\sigma)$, an agent strictly prefers investing and not participating in the intervention program. Notice when $x > \xi^* + \sigma$, $U^R(\xi^*, x) = \pi(x, 1) > \frac{s}{t}$, therefore, $\eta^*(\sigma)$ is well-defined.

Since $U^R(k, x)$ increases in x, and $U^R(\xi^*, \xi^*) = -\frac{s}{1-t} \leq \frac{s}{t} = U^R(\xi^*, \eta^*(\sigma))$, therefore, $\eta^*(\sigma) \geq \xi^*$. It immediately follows that $\lim_{\sigma \to 0} \eta^*(\sigma) = \eta \geq \xi^*$. Next, we prove $\eta = \xi^*$ by contradiction. Suppose $\eta > \xi^*$, take $\sigma \to 0$ in the left hand side of (A.38), we have

$$\lim_{\sigma \to 0} \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\eta^*(\sigma) - \theta}{\sigma}\right) \pi\left(\eta^*(\sigma), 1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right) d\theta = \pi\left(\eta, 1\right) \geq \pi(\xi^*, 1) > \frac{s}{t}$$

Contradiction to (A.38). Therefore, $\lim\limits_{\sigma \to 0} \eta^*(\sigma) = \eta = \xi^*$. ∎

Proof of Proposition 11. According to Definition 1, a partial-participation program with target ξ^* should satisfy the following conditions

1. $\pi(\underline{\theta}, 1) < -\frac{s}{1-t}$
2. $\pi(\xi^*, 1) > \frac{s}{t}$
3. $\int_0^1 \pi(\xi^*, l) dl = -\frac{s}{1-t}$

As long as the government offers (s, t) given by

$$\left(-\frac{\pi(\xi^*, 1)}{\int_0^1 \pi(\xi^*, l) dl} + 1\right)^{-1} < t < 1, \tag{A.39}$$

and

$$s = -(1 - t) \int_0^1 \pi(\xi^*, l) dl, \tag{A.40}$$

the three conditions listed above are satisfied. First, by assumption, $\pi(\underline{\theta}, 1) \leq \int_0^1 \pi(\theta_0, l) dl < \int_0^1 \pi(\xi^*, l) dl = -\frac{s}{1-t}$. Second, (A.39) can be written as $\pi(\xi^*, 1) > -\frac{1-t}{t} \int_0^1 \pi(\xi^*, l) dl = \frac{s}{t}$. Finally, the third condition directly follows equation (A.40). ∎