# Intervention with Screening in Panic-Based Runs*

Lin Shen[†] Junyuan Zou[‡]

February 28, 2022

**Abstract**

Policymakers frequently use guarantees to reduce panic-based runs in the financial system. We analyze a binary-action coordination game under the global games framework and propose a novel intervention program that screens investors based on their heterogeneous beliefs about the system's stability. This program attracts only investors who are at the margin of running, and their participation boosts all investors' confidence in the financial system. Compared with government guarantee programs, our proposed program is as effective at reducing panic runs yet features two advantages: it costs less to implement and is robust to moral hazard.

*An earlier draft of this paper was circulated under the title "Intervention with Voluntary Participation in Global Games". We are indebted to Itay Goldstein and Doron Levit for their guidance in the process. For useful comments we are grateful to Christopher Bertsch (discussant), Matthieu Bouvard (discussant), Vincent Glode, Chong Huang, Benjamin Lester, George Mailath, Stephen Morris, Christian Opp, Guillermo Ordonez, Andrew Postlewaite, Jun Qian, and Xavier Vives as well as seminar and conference participants in the Wharton Finance Seminar, INSEAD Finance Seminar, the FIRS Meeting, the WFA Meeting, the FTG Summer School, the Lisbon Meeting of Game Theory and Applications, and the Tsinghua Workshop on Theory and Finance. All errors are our own.

[†]Finance Area, INSEAD, lin.shen@insead.edu.

[‡]Finance Area, INSEAD, junyuan.zou@insead.edu.

# 1 Introduction

In the modern financial system, coordination failures among investors and financial institutions can lead to self-fulfilling panic "runs" that threaten the system's stability and generate huge welfare losses.[1] Policymakers worldwide have therefore devoted considerable effort and resources to intervention programs that aim to prevent panic among investors. For instance: during the Great Recession of 2007–2008, governments provided financial institutions with loans, explicit and implicit guarantees, and capital injections. These policies proved to be effective at restoring financial stability, but they were criticized on two grounds. First, implementing guarantee programs on such a large scale entailed large fiscal costs, which jeopardized the sustainability of sovereign debt and led – in many European countries – to a sovereign debt crisis (Acharya, Drechsler and Schnabl, 2014; Farhi and Tirole, 2018). Second, the policies were criticized for their vulnerability to moral hazard problems (Cooper and Ross, 2002; Allen et al., 2018).

Given the downsides of conventional interventions, a natural question is whether those drawbacks can be overcomed without reducing the effectiveness of measures intended to prevent self-fulfilling runs. We answer this question by analyzing a coordination game in the context of bank runs in a standard global games framework (Morris and Shin, 2003). In such an environment, even under minimal information frictions among bank investors, they hold diverse beliefs about the likelihood of bank failure. We therefore propose a type of subsidy–tax program, with voluntary participation, that screens agents based on their beliefs. Compared with conventional programs such as government guarantees, our program offers two advantages. First, only a small group of pivotal investors – who matter the most for the coordination outcome – self-select into the program, which saves on implementation costs while achieving the same effectiveness. Second, the disciplinary role of bank runs is preserved, which reduces the moral hazard problem of banks.

In the benchmark model, a continuum of investors simultaneously decide whether to run on a bank. A key aspect of this model is that the run decisions feature strategic complementarities. Since a bank fails if either its fundamentals are weak or a critical mass of investors run on the bank, an investor has more incentive to run if he expects others to do likewise. As in standard global games, each investor receives a noisy private signal about the bank's fundamentals and makes inferences about other investors' run decisions. In the unique equilibrium, investors follow a threshold strategy: they run if their private signals fall below a certain signal threshold. As a result, the bank defaults if its fundamentals fall

[1] Examples of coordination failures include – but are not limited to – bank runs (Diamond and Dybvig, 1983), financial market runs (Bernardo and Welch, 2004), debt rollover problems (Morris and Shin, 2004), and credit freezes (Bebchuk and Goldstein, 2011).

below a fundamental threshold. In a region of fundamentals just below that threshold, the bank's failure is self-fulfilling: it fails because of coordination failures among investors, and it would have survived if no investors had run on the bank. So to improve social welfare, the policymaker seeks to lower the fundamental threshold and thereby minimize self-fulfilling bank runs.

Accordingly, we allow the policymaker to offer all investors a subsidy–tax program with voluntary participation. An investor who accepts the offer and does not run on the bank will receive a subsidy if the bank fails and will pay a tax if the bank survives. We classify the responses to this intervention into three categories based on the tax-to-subsidy ratio. If a program has a high tax-to-subsidy ratio, then no investor participates and so the program has no effect on the run threshold; we refer to such programs as *zero-participation programs* (ZPPs). If a program charges a nonpositive tax, then participating investors always receive nonnegative transfers from the policymaker; hence the program attracts all investors and we call it a *full-participation program* (FPP). Many existing intervention policies, including government guarantees, benefit all investors uniformly and thus fall into that category. In this paper we propose *partial-participation programs* (PPPs), which feature positive yet low tax-to-subsidy ratios such that some but not all investors participate. Conceptually, a PPP is similar to a costly insurance policy that charges a premium if the bank survives and provides protections if it fails. More importantly, PPPs screen investors based on their beliefs about the probability of bank failure.

Under a PPP, the most optimistic investors – who view the bank's failure as unlikely – stay in the bank and decline to participate because they consider the insurance to be too expensive. In contrast, the most pessimistic investors believe that the probability of bank failure is so high that staying in the bank is not an attractive option even if they are protected by the insurance. Thus only investors with intermediate beliefs will participate in the program. From their perspective, the insurance is provided at a reasonable price and staying in the bank is attractive given the protection offered by that insurance. We show that with PPP there is a unique equilibrium in which all investors follow a threshold strategy with two thresholds. An investor will invest without participating in the program if he receives a high signal, will invest and participate in the program if his signal is intermediate (i.e., between the two thresholds), and will not invest if he receives a low signal.

Although FPPs and PPPs can each reduce coordination failures, we derive the powerful result that any PPP incurs a lower expected cost of implementation than does any FPP that achieves the same coordination outcome.[2] This result is crucial for a policymaker who has a limited budget and must allocate her funding economically among several welfare-enhancing programs. Moreover, the PPP's cost savings relative to a FPP come from both the extensive and the intensive margins. On the extensive margin, a PPP excludes the most optimistic investors; hence the policymaker subsidizes fewer investors than under a FPP. On the intensive margin, a PPP collects taxes from participants when the bank survives, which partially offsets the cost of providing subsidies in expectation.

To understand intuitively how PPPs can reduce coordination failure at a minimal cost, it is useful to go through the following thought process. Start with the original threshold equilibrium without intervention programs. In this equilibrium, investors with signals slightly below the run threshold would run on the bank. A PPP provides these investors with protection against bank failure and incentivizes them to stay in the bank. Anticipating that the introduction of a PPP will incentivize more investors to stay in the bank, all investors have greater confidence in the bank's survival. This persuades investors with even lower signals to accept the PPP offer and stay, which is again anticipated by the investors and further strengthens their incentive to stay in the bank. Repeating this thought process, the PPP-induced incentive to stay is amplified by higher-order beliefs and so coordination failures can be substantially reduced in equilibrium. At the same time, given that all investors are more optimistic about the bank's survival, the downside protection offered by the PPP becomes less appealing; hence, in equilibrium, the mass of investors who accept the offer is small.

In addition to the advantage of lower implementation costs, PPPs are also more robust (than are FPPs) to moral hazard problems. We establish this robustness by incorporating moral hazard (cf. Diamond and Rajan, 2001) into the model and then comparing government guarantee programs – the least costly FPP with zero tax and positive subsidy – with PPPs. More specifically, the banker is allowed to choose an effort level after the policymaker announces the intervention program but before investors make their run decisions. The banker enjoys a private benefit from shirking, but shirking reduces investors' payoffs from staying in the bank and so increases the likelihood of bank runs. Because the banker receives a reward if the bank survives, bank runs play a disciplinary role by increasing the cost of shirking. So without intervention programs, the monitoring effect of bank runs encourages the banker to exert a welfare-maximizing effort.

[2]The result of Diamond and Dybvig (1983) that government guarantees are costless to implement relies on the assumptions that (i) there are no information frictions and (ii) runs are driven only by panic and not by fundamentals. As long as information frictions exist, government guarantees – or any other FPPs – are costly to implement because some investors would mistakenly stay in a failing bank and ask for subsidies from the policymaker.

Government guarantee programs reduce an investor's "skin in the game" and makes his run decision insensitive to the banker's effort choice. Hence government guarantee programs hinder the monitoring role of bank runs and fail to limit shirking by the banker. In contrast, a PPP can achieve the same coordination outcome while preserving the monitoring role of bank runs. The reason is that investors internalize the cost of participating in a PPP. If the banker shirks, which reduces investors' payoffs from staying in the bank, then the tax charged by the PPP becomes so costly that no investor participates in the program. Thus the PPP acts as a switching mechanism: it deters bank runs if the banker exerts effort; but it turns into a ZPP (and so becomes inoperative) if the banker shirks. Therefore, PPPs can reduce coordination failure without inducing moral hazard.

Our proposed partial-participation program has a wide range of applications beyond bank runs; examples include credit market freezes (Bebchuk and Goldstein, 2011) and asset market freezes (Bernardo and Welch, 2004). In some of these applications, moral hazard arises not at the aggregate level but rather at the individual level. In the context of a credit freeze, for instance, banks abstain from lending in anticipation of low aggregate credit supply and low returns from lending. Government guarantees encourage bank lending. Yet they reduce banks' skin in the game and thus also their incentive to screen and monitor borrowers. Unlike the moral hazard in our benchmark model, banks' shirking in credit assessment and monitoring only reduces their own profits from lending and does not directly spill over to other banks. In other words, the moral hazard problem occurs at individual level. To demonstrate the robustness of PPPs to individual moral hazard, we modify the benchmark model and assume that an investor can earn a private benefit from shirking, which reduces the success probability of his own investment. Since FPPs (e.g., government guarantees) and PPPs both subsidize investors' losses, they lead to shirking by participating investors. However, fewer investors participate in PPPs than in FPPs. It is important to bear in mind that, for the most optimistic investors, the tax component of a PPP is so costly that they optimally decline the offer – which incentivizes them to exert effort. So under PPPs, the individual moral hazard problem is limited to medium-belief investors, the mass of whom approaches zero in the limit of vanishing information frictions. In the limit, then, there are PPPs that can restore the first-best outcome; yet no government guarantee program can do so owing to the welfare loss resulting from individual moral hazard.

The actual implementation of our proposed partial-participation program can take various forms. Besides the direct interpretation of a policymaker offering an intervention program to all investors, we examine the viability of privatizing the intervention program. We remark two caveats of privatization. First, since the provider of an intervention program must make large payments in the "run" state, investors may worry about counterparty risk when participating in a private program. Second, privatization comes with information production and revelation, which may aggravate panics and undermine the effectiveness of intervention programs.

**Literature** To the best of our knowledge, our proposed partial-participation programs that screen investors based on their beliefs is novel to the literature. In particular, we demonstrate two main advantages of such programs: cost efficiency and robustness to moral hazard. Similar to our mechanism, several papers explore policies that target a specific group of agents to reduce coordination failure. For example, Sakovics and Steiner (2012) and Choi (2014) analyze coordination games with ex ante heterogeneous agents and argue that the optimal subsidy schedule is to target a certain type of investors.[3] Our proposed PPPs contrast with these targeted intervention programs in mainly three aspects. First, we show that it is cost efficient to screen agents based on their interim beliefs rather than ex ante characteristics. In other words, policymakers should target interim rather than ex ante "pivotal" investors. Second, implementing a targeted intervention program requires the policymaker to correctly identify each investor's type, which can entail additional information acquisition costs. In contrast, our proposed PPPs incentivize "pivotal" investors to self-reveal their types, and the policymaker can simply offer a uniform intervention policy to all investors. Lastly, these papers focus on direct subsidies without a tax charge, which is a special type of the FPPs in our model. Hence the policy space that we consider is more general. In Appendix C, we compare our proposed PPPs and these targeted intervention programs in details.

Our paper also contributes to the vast literature on panic-based bank runs and policies to enhance financial stability dating back to Diamond and Dybvig (1983).[4] Although government-guarantee type of programs such as deposit insurance are proposed as effective intervention to reduce coordination failures among bank investors, the literature has also pointed out their limitations. Firstly, large-scale government guarantees link the stability of banks and sovereigns and allow bank runs to jeopardize the fiscal health of a country (Acharya, Drechsler and Schnabl, 2014; Leonello, 2018; Farhi and Tirole, 2018). Moreover, such programs are vulnerable to moral hazard problems (Cooper and Ross, 2002; Allen et al.,

[3] Similar in spirit, within the contracting literature, Segal (2003) and Bernstein and Winter (2012) analyze situations in which a principal offers contracts to a group of agents, whose trade with the principal generates externalities on one another. They show that the optimal strategy for the principal is to *divide and conquer*: subsidize some of the agents so that they take the principal's desired action; this incentivizes other agents to take the same action and allows the principal to extract more surplus from agents who are not subsidized.

[4] In an event study of early withdrawals in response to an increase in policy uncertainty in a Greek bank, Artavanis et al. (2019) quantifies that coordination motive accounts for around one-third of the total increase in early withdrawal.

2015, 2018).[5] Compared to government guarantees, our proposed program overcomes these two drawbacks without compromising the effectiveness of preventing panic runs.

In terms of methodology, our model builds on the global games literature pioneered by Carlsson and Van Damme (1993). Morris and Shin (2003) review the commonly applied set-up and applications of global games. Our benchmark model in Section 2 is a special case with binary payoffs, and we generalize the payoff structure in Appendix B. Researchers have applied global games techniques to analyze coordination failures in a variety of contexts; these include, among others, bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005), currency attacks (Morris and Shin, 1998), credit freezes (Bebchuk and Goldstein, 2011), debt rollovers (Morris and Shin, 2004; He and Xiong, 2012), and financial market runs (Bernardo and Welch, 2004; Brunnermeier and Pedersen, 2009). Although we focus on bank runs, our model has a wide range of applications. As long as the policymaker can observe agents' actions and can condition the intervention program on those actions, implementing our proposed program can reduce coordination failures.

From a theoretical perspective, the study that is the most closely related to ours is Morris and Shadmehr (2017), who analyze the reward schemes that a revolutionary leader can offer in order to elicit effort from citizens. The ideal reward scheme also screens citizens based on their beliefs. Yet they consider mandatory reward schemes, whereas we focus on intervention programs with voluntary participation. A more crucial distinction is that, while Morris and Shadmehr (2017) assume that any reward scheme can be implemented at zero cost, we explicitly model and target to minimize the cost of intervention. Two other related works are those of Cong, Grenadier and Hu (2020) and Basak and Zhou (2020), who explore intervention policies under dynamic settings. In both of these papers, the policymaker targets a subset of agents in each period; successful coordination in previous periods serves as a public signal that upholds agents' confidence in subsequent periods. Our paper considers a static coordination game, in which investors' confidence is boosted by their inferences about other investors' reactions to an intervention program. Therefore, the effectiveness of our proposed intervention programs does not rely on a dynamic information structure. Another related paper is Bond and Hagerty (2010). In the context of crime waves, they analyze the welfare implications of introducing a medium action of moderate crime with mild penalty if detected. Our proposed PPPs are similar in the sense that they also introduce a medium action with moderate benefit and mild cost. The difference is that they focus on the distinct welfare implications in different equilibria, while we focus on the amplification effect through

[5] See Demirgüç-Kunt and Detragiache (2002); Demirgüç-Kunt and Huizinga (2004); Ioannidou and Penas (2010) for empirical evidence that deposit insurance reduces monitoring by bank investors and increases bank risk-taking.

higher-order beliefs when agents have asymmetric information about the state of the world.

The rest of our paper proceeds as follows. In Section 2, we present a benchmark model of a binary-action investment game and introduce intervention policies that can reduce coordination failures. Sections 3 and 4 compare the proposed program with government guarantees in terms of implementation costs and robustness to moral hazard respectively. Section 5 discusses issues related to implementation of our proposed program. We conclude in Section 6 with a brief summary of our study's results and limitations.

# 2 Benchmark Model

In this section, we analyze a benchmark model in the context of classic panic-based bank runs. We first describe the environment without intervention programs and show that coordination failure among investors can lead to welfare losses. Then we introduce intervention programs and demonstrate how they can help reduce the likelihood of such failure.

## 2.1 Bank Runs

There are three periods, $t \in \{0, 1, 2\}$. At $t = 0$, a financial institution (for short, a bank) collects one unit of capital from a unit mass of infinitesimal investors, indexed by $i \in [0, 1]$, in the form of demandable debt. The bank then makes long-term investments that mature at $t = 2$ and can be liquidated prematurely, at a cost, at $t = 1$.[6]

At $t = 1$, each investor $i$ makes a withdrawal decision $a_i \in \{0, 1\}$: $a_i = 0$ if investor $i$ withdraws his funds early, or $a_i = 1$ if he stays in the bank until $t = 2$. To accommodate early withdrawals, the bank must undertake a costly liquidation, at $t = 1$, of its long-term investments. A bank fails if it has insufficient assets to fulfill early withdrawals. Investors who withdraw their funds early ($a_i = 0$) are guaranteed to receive back their initial investments. Investors who stay ($a_i = 1$) receive nothing if the bank fails but enjoy returns of $R > 1$ if the bank survives. In what follows, we normalize investor $i$'s payoff from early withdrawal to 0. Hence his payoff from staying is

$$\pi(\theta, l) = \begin{cases} R - 1 & \text{if } 1 - l \le \theta, \\ -1 & \text{if } 1 - l > \theta. \end{cases} \tag{1}$$

Here $1 - l = \int_0^1 \mathbb{1}_{\{a_i = 0\}} \, \mathrm{d}i$ is the aggregate early withdrawal at $t = 1$, and $\theta$ represents the fundamentals that determine the maximum amount of early withdrawals that the bank can

[6]Besides financial institutions, the environment described here encompasses any firm that relies on short-term debt.

fulfill. Economically, $\theta$ reflects the quality of the bank's assets as well as the market conditions under which the bank liquidates its long-term investments. When the bank experiences total early withdrawals $1-l$ in excess of its capacity (as determined by the fundamentals $\theta$), the bank fails.

In the benchmark model we focus, for the sake of clarity, on a simple binary payoff structure.[7] In Appendix B, we consider a generalized continuous payoff structure and demonstrate the robustness of our main results to that alternative. It is important that the payoff structure of investors features strategic complementarities (as in Diamond and Dybvig, 1983); hence an investor's incentive to withdraw his funds early increases with the mass of other investors who do so.

In terms of information structure, we follow the standard global games literature and make the following assumptions. At $t=0$, the fundamentals term $\theta$ is drawn from a uniform distribution with support $[\underline{\theta}, \bar{\theta}]$ and is not observable to investors.[8] At $t=1$, each investor $i$ receives a private and noisy signal, $x_i = \theta + \sigma\varepsilon_i$, about the fundamentals; here $\sigma$ represents the magnitude of information friction and $\varepsilon_i$ is independent and identically distributed (i.i.d.) with a continuous and strictly increasing cumulative distribution function $F(\varepsilon)$, the support of which is $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Furthermore, we assume that $\underline{\theta} < -\sigma$ and $\bar{\theta} > 1+\sigma$. Under these assumptions, there exist two dominance regions of signals, $\left[-\underline{\theta} - \frac{1}{2}\sigma, \underline{x}\right)$ and $\left(\bar{x}, \bar{\theta} + \frac{1}{2}\sigma\right]$. Here $\underline{x}$ and $\bar{x}$ are such that

$$\Pr[\theta \geq 1 \mid x = \bar{x}] = \frac{1}{R},$$
$$\Pr[\theta \geq 0 \mid x = \underline{x}] = \frac{1}{R}.$$

We can see intuitively that an investor who receives a *high* private signal $\bar{x}$ is indifferent between running or not, even when all other investors run on the bank ($l=0$). So if an investor receives a signal $x > \bar{x}$ then his dominant strategy is to stay in the bank. An investor who receives a *low* signal $\underline{x}$ is, analogously, indifferent between running or not, even when all other investors stay in the bank ($l=1$). In this case, if $x < \underline{x}$ then withdrawing early is the dominant strategy.

[7]The payoff structure we adopt resembles the set-up of Rochet and Vives (2004); in that study, fund managers – just like the bank investors in our model – make run decisions and earn binary payoffs.

[8]We assume a uniform prior in order to obtain an analytical solution to the coordination game. It can be viewed as an approximation of a continuous prior distribution function when the magnitude of information friction is small.

## 2.2 Equilibrium without Intervention

We now analyze the equilibrium without intervention and identify the inefficiencies due to coordination failure. Proposition 1 characterizes the equilibrium.

**Proposition 1** *Without intervention, there is a unique equilibrium in which all agents follow the same strategy:*

$$a_i(x_i) = \begin{cases} 1 & \text{if } x_i \geq \xi_0^*, \\ 0 & \text{if } x_i < \xi_0^*; \end{cases}$$

*here* $\xi_0^* = \frac{1}{R} + \sigma F^{-1}\left(\frac{1}{R}\right)$.

Given any realization of $\theta$, we can use this proposition to calculate the total early withdrawal $1-l$ and thus to predict the coordination outcomes. Because all investors follow the same threshold strategy in equilibrium, the coordination outcome also features a threshold above which the bank survives. Let $\theta^*(\xi)$ denote the fundamental threshold when all agents follow the threshold strategy $\xi$; then $\theta^*(\xi)$ is defined by

$$F\left(\frac{\xi - \theta^*(\xi)}{\sigma}\right) = \theta^*(\xi). \tag{2}$$

That is to say: at the fundamental threshold, the amount of early withdrawals reaches the bank's maximum capacity. So in equilibrium, the fundamental threshold is given by

$$\theta^*(\xi_0^*) = \frac{1}{R}.$$

We can therefore divide the fundamental realizations into three regions, as shown in Figure 1.


Figure 1: Coordination Outcome

In the middle region $\theta \in [0, 1/R)$, if no investor withdraws his funds early then the bank survives. Yet in equilibrium, enough investors form self-fulfilling beliefs that others will run on the bank, and the bank indeed suffers severe runs and fails. In this region, a unit mass of early withdrawals leads to a social welfare loss of $R - 1 > 0$. Hence the first-best scenario is for all investors to stay in the bank as long as $\theta \geq 0$. We shall next introduce intervention programs and show how they lower the fundamental threshold, which reduces the inefficiencies caused by coordination failure among investors.

## 2.3 Intervention Programs

Having characterized the equilibrium in the game without intervention, we are prepared to describe the subsidy–tax intervention program that a policymaker can use to minimize panic-based bank runs.

At $t = 0$, the policymaker announces an intervention program $(s, t)$ before the realization of fundamentals $\theta$. The program is offered only to investors who choose to stay in the bank, and their participation in the program is voluntary.[9] At $t = 1$, investors observe their respective private signals $x_i$ and decide whether to withdraw their funds early – and, if not, whether to accept or decline the offer $(s, t)$. Investor $i$'s action space is then enlarged to $a_i \in \{0, 1_A, 1_D\}$: $a_i = 0$ if investor $i$ withdraws his funds early; $a_i = 1_A$ if he stays in the bank and accepts the offer; and $a_i = 1_D$ if he stays in the bank and declines the offer. When $a_i = 1_D$ or $a_i = 0$, investor $i$'s payoff remains as specified in Section 2.1. In particular, the payoff from early withdrawal ($a_i = 0$) is simply zero and the payoff from staying and declining the offer ($a_i = 1_D$) is

$$\pi_D(\theta, l) = \pi(\theta, l).$$

When $a_i = 1_A$, investor $i$ receives a subsidy $s$ if the bank fails or must pay a tax $t$ if the bank survives. The expected payoff from staying and accepting the offer ($a_i = 1_A$) can therefore be written as

$$\pi_A(\theta, l) = \begin{cases} R - 1 - t & \text{if } 1 - l \leq \theta, \\ -1 + s & \text{if } 1 - l > \theta. \end{cases}$$

In essence, the offer $(s, t)$ amounts to a costly insurance; $t$ is the insurance premium and $s + t$ is the coverage when the bank fails. The subsidy $s$ reduces agents' exposure to the risk of a coordination failure and encourages them to stay in the bank. The tax $t$ serves two purposes. First, as we show in Section 3, it reduces the cost of implementing the intervention program by directly collecting taxes when the bank survives and indirectly discouraging optimistic agents from participating in the program. Second, as established in Section 4, the tax deters moral hazard problems by keeping each participating investor's skin in the game. Because investors receive binary payoffs, it is sufficient to focus on intervention programs $(s, t)$ involving lump-sum transfers between the policymaker and participating investors.[10] In

[9] An implicit assumption here is that the policymaker can observe and contract on investors' actions. As shown in Bond and Pande (2007), if the policymaker cannot observe individual actions then her ability to use subsidy–tax schemes as a coordination device is extremely limited. One implication is that the intervention program discussed here cannot be applied to currency attacks (Morris and Shin, 1998), in which the actions of agents are difficult to trace.

[10] In Appendix B we generalize the payoff structure to be continuous. In a similar spirit, the intervention program then consists of a lump-sum subsidy and a tax that is proportional to investors' payoffs.

what follows, we restrict $s \geq 0$ and $t \leq R$ because of investors' limited liability. In addition, we also impose $s \leq 1$ since intervention programs with $s > 1$ incentivize investors to say in the bank even when it fails for sure ($\theta < 0$).

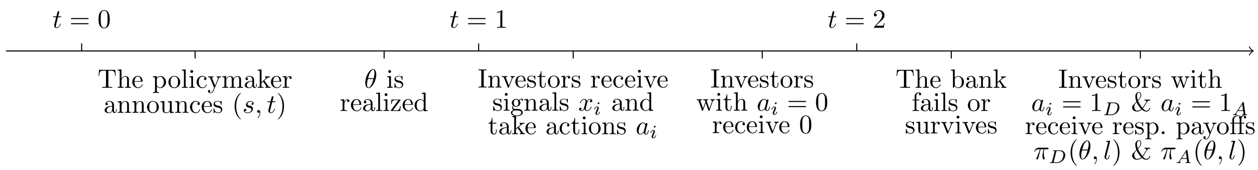



Figure 2: Timeline

Figure 2 summarizes the sequence of events in the game with an intervention program. We can immediately see one advantage of the intervention program $(s, t)$: it does not depend on the fundamentals $\theta$. Angeletos, Hellwig and Pavan (2006) point out the informational effect of state-contingent policies – namely, they can reveal the policymaker's private information about the state of the economy. This informational effect not only undermines the efficiency of intervention policies but also leads to multiple equilibria in global games. Yet the intervention programs analyzed in this paper are free of such problems, since the announcement of a program does *not* convey any information about $\theta$ that the policymaker may possess. Moreover, a policymaker can implement the program without having any superior information about the fundamentals $\theta$.

Even though intervention program $(s, t)$ is specified in a subsidy–tax form, it can be interpreted as other forms of intervention with transfers – contingent on the coordination result – between policymaker and investors. For example, a government guarantee–type program that promises to cover bank failure losses up to $s$ is equivalent to a subsidy–tax program $(s, 0)$.

## 2.4 Equilibrium with Intervention

We now analyze the equilibrium with intervention and demonstrate how intervention programs reduce coordination failure. Under intervention program $(s, t)$, each investor $i$ has three choices: $a_i \in \{0, 1_A, 1_D\}$. Note that investors' participation decisions – whether $a_i = 1_D$ or $1_A$ – have no impact on the aggregate early withdrawal $1 - l = \int_0^1 \mathbb{1}_{\{a_i = 0\}} \,\mathrm{d}i$ or on the bank's survival. As a result, when analyzing an investor's equilibrium strategies it is sufficient to condition on *other* investors' withdrawal strategies. We facilitate discussion by defining $p_i$, the interim belief of investor $i$, as his estimate of the probability that the bank will survive; that estimate is conditional on (a) his private signal $x_i$ and (b) other agents' strategies $a_{-i}(x)$:

$$p_i = \Pr[1 - l \leq \theta \mid x_i; a_{-i}(x)].$$

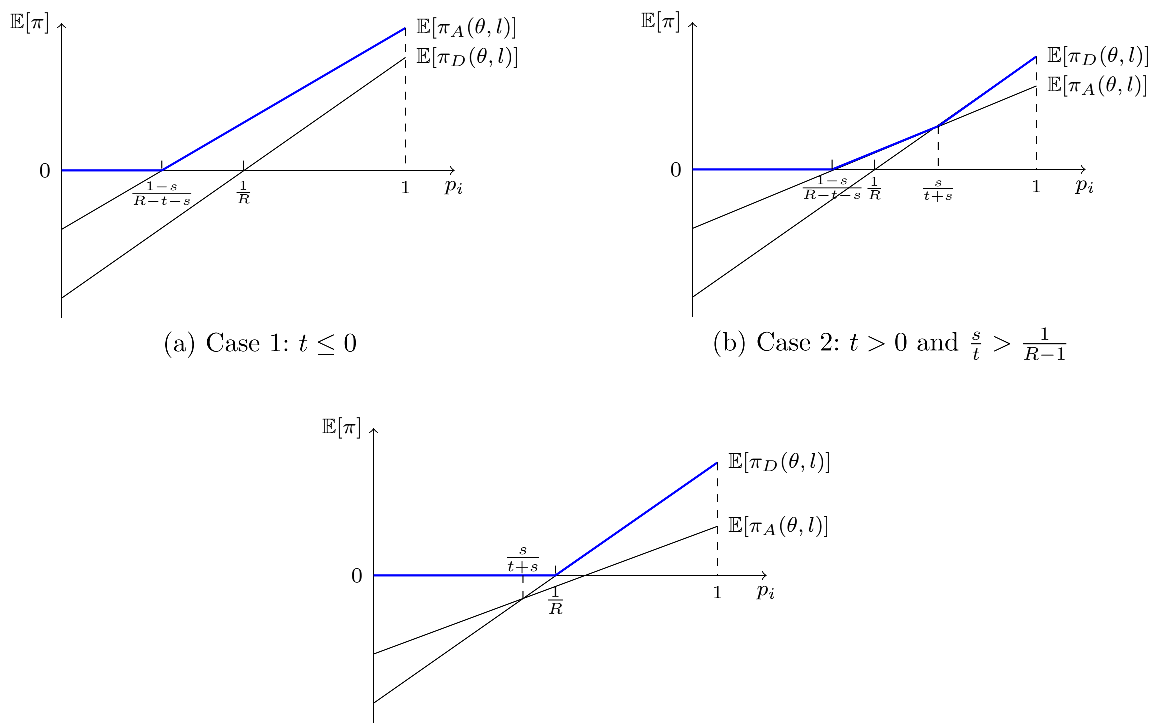
Given his belief, investor $i$'s expected payoffs from $a_i = 1_D$ and $a_i = 1_A$ are (respectively)

$$\mathbb{E}[\pi_D(\theta, l) \mid x_i; a_{-i}(x)] = p_i R - 1 \quad \text{and} \tag{3}$$

$$\mathbb{E}[\pi_A(\theta, l) \mid x_i; a_{-i}(x)] = p_i(R - t - s) - (1 - s); \tag{4}$$

his expected payoff from early withdrawal ($a_i = 0$) is zero.

Figure 3 plots expected payoffs as a function of the interim belief $p_i$ for three action choices; in each graph, the blue line corresponds to the maximum payoff. The three cases reflect distinct levels of the intervention program's generosity.



(a) Case 1: $t \leq 0$

(b) Case 2: $t > 0$ and $\frac{s}{t} > \frac{1}{R-1}$

(c) Case 3: $t > 0$ and $\frac{s}{t} \leq \frac{1}{R-1}$

Figure 3: Expected Payoffs and Interim Beliefs

In the first case, shown in panel (a), if $t \leq 0$ then the intervention program is a "free lunch". That is, an investor who accepts the offer will receive a nonnegative transfer from the policymaker regardless of the coordination outcome. It follows that accepting dominates declining, so all investors who stay in the bank will accept the offer; thus we have a so-called *full-participation program* (FPP). Absent intervention, the belief threshold for early withdrawal is $\frac{1}{R}$, at which point an investor is indifferent between withdrawing early or not.

With a full-participation program, the threshold falls to $\frac{1-s}{R-t-s}$. In the third case, illustrated in panel (c) of the figure, if $t > 0$ and $\frac{s}{t} \leq \frac{1}{R-1}$ then declining the offer dominates accepting it. We call this a *zero-participation program* (ZPP). Here the threshold belief with intervention is simply the same as without intervention: $\frac{1}{R}$.

The most interesting case is shown in Figure 3(b). Here $t > 0$ and $\frac{s}{t} > \frac{1}{R-1}$, so an investor who participates in the program is essentially purchasing insurance against bank failure. If the bank survives, a participating investor is taxed an amount $t$; if the bank fails, the investor receives compensation $s$. For investors with extreme beliefs, the insurance is not valuable. On the one hand, investors with optimistic beliefs $p_i > \frac{s}{t+s}$ estimate a high probability of bank survival and so they would stay in the bank even without the intervention program. Furthermore, given the high probability of paying $t$ and low probability of receiving $s$, this "insurance" (intervention program) is overpriced. Therefore, the optimal action of such an investor is to stay in the bank and decline the offer. On the other hand, investors with pessimistic beliefs $p_i < \frac{1-s}{R-t-s}$ estimate a high probability of bank failure. As long as the intervention program does not guarantee zero loss upon bank failure ($s < 1$), the "insurance" does not provide enough coverage for them to stay in the bank. Hence their optimal action for such investors is to run on the bank ($a_i = 0$).

In contrast, the insurance generates value for investors who have medium beliefs $p_i \in \left[\frac{1-s}{R-t-s}, \frac{s}{t+s}\right)$ because they are uncertain about the coordination outcome. Since not all staying investors participate in the program, we call this type of program a *partial-participation program* (PPP). It is worth noting that, in effect, partial-participation programs convince investors to stay in the bank when their beliefs $p_i \in \left[\frac{1-s}{R-t-s}, \frac{1}{R}\right]$ – that is, investors who would run on the bank *without* an intervention program. As a consequence, the threshold belief is lowered to $\frac{1-s}{R-t-s}$. Both full- and partial-participation programs lower the threshold belief and discourage bank runs; yet as we show in Section 3, these two program types differ substantially in implementation costs.

Our next proposition characterizes the equilibrium in the presence of a subsidy–tax intervention program $(s, t)$.

**Proposition 2** *When the policymaker offers a subsidy–tax intervention program $(s, t)$ with $s \in [0, 1]$ and $t \leq R$, the game has a unique equilibrium.*

1. *(Full-participation programs) If $t \leq 0$, then the equilibrium stategy of investor $i$ is*

$$a_i = \begin{cases} 1_A & \textit{if } x_i \geq \xi^*(s,t), \\ 0 & \textit{if } x_i < \xi^*(s,t), \end{cases}$$

*where*

$$\xi^*(s,t) = \frac{1-s}{R-t-s} + \sigma F^{-1}\left(\frac{1-s}{R-t-s}\right).$$

2. *(Partial-participation programs) If $t > 0$ and $\frac{s}{t} > \frac{1}{R-1}$, then the equilibrium stategy of investor $i$ is*

$$a_i = \begin{cases} 1_D & \text{if } x_i \geq \eta^*(s,t), \\ 1_A & \text{if } \xi^*(s,t) \leq x_i < \eta^*(s,t), \\ 0 & \text{if } x_i < \xi^*(s,t); \end{cases}$$

*here $\eta^*(s,t)$ is the participation threshold and*

$$\xi^*(s,t) = \frac{1-s}{R-t-s} + \sigma F^{-1}\left(\frac{1-s}{R-t-s}\right),$$
$$\eta^*(s,t) = \frac{1-s}{R-t-s} + \sigma F^{-1}\left(\frac{s}{t+s}\right).$$

3. *(Zero-participation programs) If $t > 0$ and $\frac{s}{t} \leq \frac{1}{R-1}$, then the equilibrium stategy of investor $i$ is*

$$a_i = \begin{cases} 1_D & \text{if } x_i \geq \xi^*(s,t), \\ 0 & \text{if } x_i < \xi^*(s,t), \end{cases}$$

*where*

$$\xi^*(s,t) = \xi_0^* = \frac{1}{R} + \sigma F^{-1}\left(\frac{1}{R}\right).$$

The proof of Proposition 2 is given in Appendix D. For FPPs (case 1) and ZPPs (case 3), staying investors either all accept or all decline the offer. Therefore, the equilibrium analyses directly follow Proposition 1. It is interesting that investors' responses to PPPs (case 2) are nontrivial. Recall that such programs incentivize investors with beliefs $p_i \in \left[\frac{1-s}{R-t-s}, \frac{1}{R}\right]$ to stay in the bank, reducing the aggregate early withdrawal $1-l$. In turn, this reduction in $1-l$ strengthens the incentive of all investors to stay in the bank; that outcome further incentivizes investors to stay and thus lowers $1-l$. This virtuous cycle amplifies the effectiveness of PPPs at reducing coordination failure. Next we outline the equilibrium analyses and discuss the intuition for this amplification effect.

It turns out that we can focus on threshold strategies.[11] Investor $i$ cares only about whether other investors run on the bank; he is not concerned about whether they participate in the intervention program. We therefore use the language that another investor $j$ follows a *threshold run strategy* with threshold $k$ provided he runs on the bank ($a_j = 0$) if and only

[11]See the proof of Proposition 2 for details.

if $x_j < k$. Then we can rewrite the interim belief $p_i = p(x_i; k)$ as a function of investor $i$'s private signal $x_i$ and the run threshold for all other investors $k$:

$$p(x_i; k) = \Pr[\theta > \theta^*(k) \mid x_i] = F\left(\frac{x_i - \theta^*(k)}{\sigma}\right), \tag{5}$$

where $\theta^*(k)$ is the fundamental threshold for bank survival and satisfies $F\left(\frac{k-\theta^*(k)}{\sigma}\right) = \theta^*(k)$ as defined in (2). Observe that $p(x_i; k)$ increases with $x_i$ but decreases with $k$. It makes sense that (a) a high private signal $x_i$ indicates a high realization of fundamentals $\theta$ and (b) a low run threshold $k$ is suggestive of a low aggregate early withdrawal $1 - l$; both imply a high likelihood of bank survival.

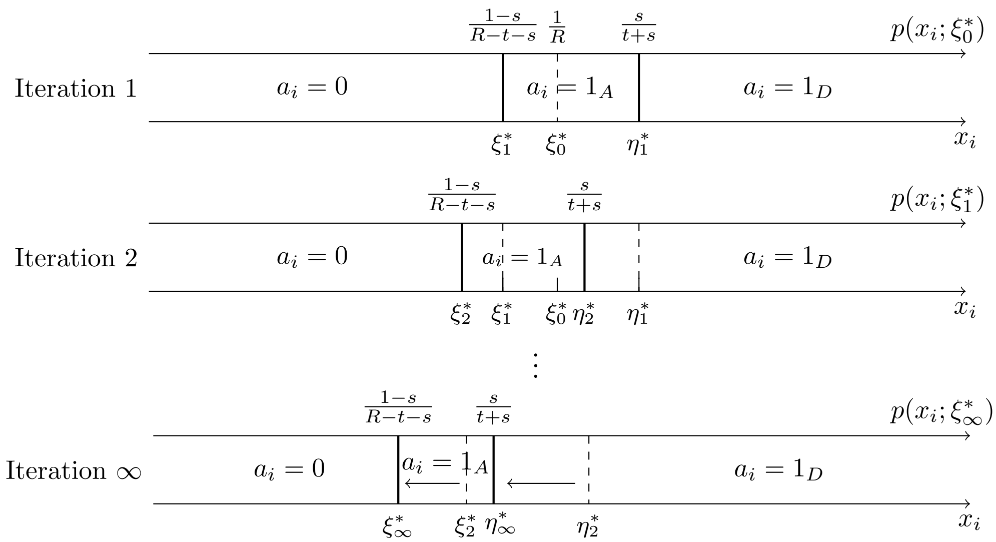



Figure 4: Amplification via Higher-Order Beliefs

Figure 4 illustrates how the effectiveness of PPPs is amplified by investors' higher-order beliefs. In each iteration, the lower and upper axes represent the signal received by an investor and his corresponding belief respectively. Start from the threshold run strategy $\xi_0^*$, which is the equilibrium threshold without intervention. An investor who believes that other investors all adopt run threshold $\xi_0^*$ is willing to lower his own run threshold to $\xi_1^*$ by participating in the intervention program. So in iteration 2, investors update their beliefs knowing that other investors have adopted a lower run threshold $\xi_1^* < \xi_0^*$. Since $p(x_i; k)$ decreases with $k$, it follows that all investors – given their private signals fixed – become more optimistic in iteration 2 about bank survival. Intuitively, all agents realize the intervention program will incentivize more investors to stay, which reduces early withdrawals $l$ and stabilizes the bank; hence they are willing to lower their run threshold further, to $\xi_2^*$. Similarly, in iteration 3, all investors become more optimistic because they are aware that investors with signals between

$\xi_1^*$ and $\xi_2^*$ are incentivized to stay in the bank; therefore, investors lower their investment threshold further to $\xi_3$, and so forth. Yet as investors become more optimistic about the bank's stability, the intervention program becomes less appealing, which implies a decreasing sequence of participation thresholds $\{\eta_n^*\}_{n=1}^{\infty}$. With an infinite number of iterations, both the investment threshold and the participation threshold decrease markedly in equilibrium. Hence the mass of investors accepting the offer becomes fairly small while the probability of bank runs is appreciably reduced. We call these program participants *pivotal* investors because their payoffs determine the equilibrium fundamental threshold $\theta^*$.

It is worth mentioning that, investors are *ex ante* homogeneous in our model, and they receive heterogeneous private signals in the *interim* stage. In fact, the intuition for the effectiveness of PPPs still holds when investors are ex ante heterogeneous. As we demonstrate in Appendix C, with ex ante heterogeneous investors, PPPs screen for pivotal investors with medium beliefs within *each* ex ante type. Interestingly and importantly, this insight contrasts that in Sakovics and Steiner (2012), who identify ex ante pivotal types of investors and propose type-specific subsidy programs targeting at these ex ante pivotal investors. We show that PPPs are more cost-efficient as they screen for interim rather than ex ante "pivotal" investors. Moreover, unlike type-specific subsidy programs in Sakovics and Steiner (2012), PPPs do not require regulators to observe and identify investors' ex ante characteristics.

Both full- and partial-participation programs achieve a fundamental threshold above which the bank survives,

$$\theta^*(\xi^*(s,t)) = \frac{1-s}{R-t-s}, \tag{6}$$

which is lower than the threshold $\theta^*(\xi_0^*)$ *without* intervention. Therefore, offering this intervention program successfully reduces the region of inefficient bank runs. If the policymaker sets $s = 1$ and $t < R - 1$ then the fundamental threshold can be reduced to zero, which entirely eliminates the region of inefficient bank runs; see Figure 5.

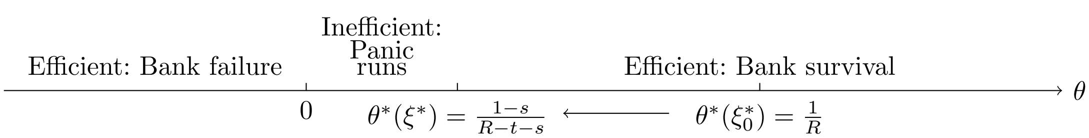



Figure 5: Coordination Outcome after Intervention

# 3 Cost of Intervention

In this section, we compare the welfare achieved by different intervention programs and show that partial-participation programs dominate full-participation programs in two steps. In Section 3.1, we define the policymaker's welfare maximization problem. The key step in solving the policymaker's problem is to find a cost-minimizing intervention program that achieves a socially optimal fundamental threshold. In Section 3.2, we prove that achieving any target fundamental threshold always incurs a lower implementation cost under a PPP than under a FPP. We further characterize the PPP with the lowest implementation cost.

## 3.1 Welfare Maximization

With intervention program $(s,t)$, social welfare is equal to investors' total investment payoffs before tax and subsidy *minus* the cost of implementing the intervention program.

Following realization of the fundamentals $\theta$, investors' total payoffs before tax and subsidy can be written as

$$W(\theta,\hat{\theta};\sigma) = \begin{cases} -\left[1-F\left(\frac{\hat{\theta}+\sigma F^{-1}(\hat{\theta})-\theta}{\sigma}\right)\right] & \text{if } \theta < \hat{\theta}, \\ (R-1)\left[1-F\left(\frac{\hat{\theta}+\sigma F^{-1}(\hat{\theta})-\theta}{\sigma}\right)\right] & \text{if } \theta \geq \hat{\theta}, \end{cases} \tag{7}$$

where $\hat{\theta} = \theta^*(\xi^*(s,t))$ is the equilibrium fundamental threshold implemented by intervention program $(s,t)$. Equation (7) implies that the equilibrium fundamental threshold is a sufficient statistic for investors' payoffs. In other words, if two intervention programs implement the same fundamental threshold, investors enjoy the same payoffs before tax and subsidy under the two intervention programs. Moreover, as established in Proposition 3 below, any fundamental threshold $\hat{\theta} \in [0, 1/R)$ is implementable by both FPPs and PPPs. Hence the policymaker can maximize social welfare by identifying the least costly program $(s,t)$ that implements a socially optimal fundamental threshold $\hat{\theta}$.

**Proposition 3** *Given any $\sigma > 0$ and any fundamental threshold $\hat{\theta} \in [0, 1/R)$, there exists a non-empty set of FPPs $\left\{(s,t) : R - \frac{1}{\hat{\theta}} \leq t \leq 0, s = \frac{\hat{\theta}t+1-\hat{\theta}R}{1-\hat{\theta}}\right\}$ and a non-empty set of PPPs $\left\{(s,t) : 0 < t < R-1, s = \frac{\hat{\theta}t+1-\hat{\theta}R}{1-\hat{\theta}}\right\}$ that implement $\hat{\theta}$.*

Proposition 3 characterizes all the intervention programs – both FPPs and PPPs – that implement a given target $\hat{\theta} \in [0, 1/R)$. Note that given $t$, there is a one-to-one mapping between $\hat{\theta}$ and $s$. Therefore, an intervention program $(s,t)$ can be fully characterized by its tax charge $t$ and implemented fundamental threshold $\hat{\theta}$. For the rest of this section, we use

$(\hat{\theta}, t)$ to denote an intervention program $(s, t)$ that satisfies

$$s(\hat{\theta}, t) = \frac{\hat{\theta}t + 1 - \hat{\theta}R}{1 - \hat{\theta}}. \tag{8}$$

The relevant parameter regions are $0 \leq \hat{\theta} \leq \theta^*(\xi_0^*) = \frac{1}{R}$ and $R - \frac{1}{\hat{\theta}} \leq t < R - 1$.[12]

Below we formally characterize the policymaker's welfare maximization problem. In particular, the policymaker chooses an intervention program $(\hat{\theta}, t)$ to maximize investors' total payoffs minus the cost of implementation:

$$\max_{0 \leq \hat{\theta} \leq \frac{1}{R},\ R - \frac{1}{\hat{\theta}} \leq t < R-1} \mathbb{E}_\theta[W(\theta, \hat{\theta}; \sigma)] - \mathbb{E}_\theta[C(\theta, \hat{\theta}, t; \sigma)].$$

Here $C(\theta, \hat{\theta}, t; \sigma)$ is the ex post welfare cost of providing intervention program $(\hat{\theta}, t)$ after the realization of fundamentals $\theta$. We will specify $C(\theta, \hat{\theta}, t; \sigma)$ in Section 3.2.

The problem can be solved in two steps. First, for any target fundamental threshold $\hat{\theta}$, the policymaker selects the least costly program $(\hat{\theta}, t)$ that implements $\hat{\theta}$:

$$C_{min}(\hat{\theta}; \sigma) = \inf_{R - \frac{1}{\hat{\theta}} \leq t < R-1} \mathbb{E}_\theta[C(\theta, \hat{\theta}, t; \sigma)].$$

Second, the policymaker chooses the optimal target $\hat{\theta}$ to maximize social welfare:

$$\max_{0 \leq \hat{\theta} \leq \frac{1}{R}} \mathbb{E}_\theta[W(\theta, \hat{\theta}; \sigma)] - C_{min}(\hat{\theta}; \sigma).$$

In Section 3.2, we focus on the first step – that is, the cost minimization problem of achieving a target value of $\hat{\theta}$. As we will show, PPPs always incur a lower cost than do FPPs. It follows that the optimal intervention program must be a partial-participation program regardless of the optimal target $\hat{\theta}$. In Appendix A, we complete the analysis and provide comparative statics of the optimal target $\hat{\theta}$ with respect to parameters of the model.

## 3.2 Cost Minimization

In this section, we first specify the welfare cost of implementation $C(\theta, \hat{\theta}, t; \sigma)$. After that, we compare the expected cost of all programs that implement a given fundamental thresh-

[12] For completeness, the "laissez-faire" threshold $\hat{\theta} = \theta^*(\xi_0^*) = 1/R$ is implementable by any zero-participation program with zero cost of implementation. Here we exclude those zero-participation programs $(s, t)$ that doesn't satisfy the constraints: $s = \frac{t}{R-1}$ and $t < R - 1$ to preserves the consistency of notation $(\hat{\theta}, t)$ at $\hat{\theta} = 1/R$.

old $\hat{\theta}$. Finally, we characterize the least costly intervention program to implement $\hat{\theta}$ and the corresponding minimal cost function $C_{min}(\hat{\theta};\sigma)$.

**Specification of cost function** To specify the welfare cost of implementation $C(\theta,\hat{\theta},t;\sigma)$, it is convenient to first analyze the *net transfer* from the policy maker to participating investors. With intervention program $(\hat{\theta},t)$, i.e. an intervention program $(s,t)$ that satisfies Equation (8), the net transfer to *each* participating investor in state $\theta$ is

$$\gamma(\theta,\hat{\theta},t)=\begin{cases}-t & \text{if } \theta\geq\hat{\theta},\\ s(\hat{\theta},t) & \text{if } \theta<\hat{\theta}.\end{cases} \tag{9}$$

Note that $\gamma(\theta,\hat{\theta},t)$ speaks to net transfer on the *intensive margin*. On the *extensive margin*, the mass of participating investors in state $\theta$ – which we denote as $P(\theta,\hat{\theta},t;\sigma)$ – depends (according to Proposition 2) on the type of program:

$$P(\theta,\hat{\theta},t;\sigma)=\begin{cases}1-F\left(\frac{\xi^*(s(\hat{\theta},t),t)-\theta}{\sigma}\right) & \text{if } R-\frac{1}{\hat{\theta}}\leq t\leq 0,\\ F\left(\frac{\eta^*(s(\hat{\theta},t),t)-\theta}{\sigma}\right)-F\left(\frac{\xi^*(s(\hat{\theta},t),t)-\theta}{\sigma}\right) & \text{if } 0<t<R-1.\end{cases} \tag{10}$$

As specified in Proposition 3, a target $\hat{\theta}\in[0,1/R)$ can be implemented by both FPPs and PPPs. If $R-\frac{1}{\hat{\theta}}\leq t\leq 0$ then we have a FPP: all investors with a private signal that exceeds the run threshold participate in the intervention program. If $0<t<R-1$ then we have a PPP, where only pivotal investors with private signals between the run and participation thresholds participate.

The total net transfer is simply the transfer per participant *multiplied by* the mass of participants:

$$\Gamma(\theta,\hat{\theta},t;\sigma)=\gamma(\theta,\hat{\theta},t)P(\theta,\hat{\theta},t;\sigma). \tag{11}$$

To analyze the *welfare* cost of implementing an intervention program $(\hat{\theta},t)$, we assume that each dollar transferred from the policymaker to investors incurs a $\tau$ unit of welfare loss. Here $\tau$ should not be interpreted as monetary value of the transfer, instead $\tau$ represents the policymaker's opportunity cost of forgoing other welfare-enhancing projects, and each dollar in the budget of the policymaker can be used to generates a welfare gain of $\tau$.[13] Hence the welfare cost of offering intervention program $(\hat{\theta},t)$ in state $\theta$ is

$$C(\theta,\hat{\theta},t;\sigma)=\tau\Gamma(\theta,\hat{\theta},t;\sigma). \tag{12}$$

[13] We can think of a benevolent policymaker allocating her limited budget to maximize social welfare, in which case $\tau$ corresponds to the "shadow value" of her budget constraint.

As a benchmark, if $\tau = 0$, the expected welfare cost of implementing any program is zero. In this case, any two programs that implement the same fundamental threshold $\hat{\theta}$ achieve the same expected welfare in equilibrium. This is an ideal case when the policymaker has abundant budget and transferring resources to investors incurs no opportunity cost, and so $C(\theta, \hat{\theta}, t; \sigma) = 0$. In what follows, we assume $\tau > 0$, and it is optimal to implement $\hat{\theta}$ with the least costly program, or equivalently the program with the lowest expected net transfer.

**Comparison of PPPs and FPPs** Proposition 4 compares the expected net transfers of all intervention programs – including both FPPs and PPPs – that achieve the same target threshold $\hat{\theta}$. The comparison in terms of implementation cost follows directly as it is proportional to net transfers.

**Proposition 4** *Given any $\sigma > 0$ and any target fundamental threshold $\hat{\theta} \in [0, 1/R)$, the expected net transfer $\mathbb{E}_\theta \left[\Gamma(\theta, \hat{\theta}, t; \sigma)\right]$ is continuous and strictly decreases in $t$.*

This proposition delivers a general and powerful result that the expected net transfer to implement $\hat{\theta}$ decreases in tax charge $t$. Here we delineate the intuition underlying this result while leaving the proof to Appendix D. Suppose the regulator raises the tax charge to achieve the same target fundamental threshold $\hat{\theta}$. That is, she switches from intervention program $(\hat{\theta}, t)$ to $(\hat{\theta}, t + \Delta t)$ with $\Delta t > 0$. To compensate the investors, she has to also raise the subsidy, i.e. $\Delta s \equiv s(\hat{\theta}, t + \Delta) - s(\hat{\theta}, t) = \frac{\hat{\theta}}{1-\hat{\theta}}\Delta t > 0$, which directly follows Equation (8). In other words, the new program is a more expensive insurance with higher coverage. Proposition 4 implies that the new program reduces the expected net transfer. In fact, such reduction stems from both intensive and extensive margins.

On the *intensive margin*, the expected net transfer to each participating investor is lower under the new program. Consider the interim belief of an investor with a private signal $x_i$:

$$p_i = \Pr[\theta \geq \hat{\theta} \mid x_i] = F\left(\frac{x_i - \hat{\theta}}{\sigma}\right). \tag{13}$$

Because the fundamental threshold $\hat{\theta}$ remains the same, investors with the same private signal $x_i$ form the same belief $p_i$. For an investor with a private signal $x_i$ and belief $p_i$, the additional transfer he expects to receive under the new program is

$$\mathbb{E}_\theta[\gamma(\theta, \hat{\theta}, t + \Delta t)|x_i] - \mathbb{E}_\theta[\gamma(\theta, \hat{\theta}, t)|x_i] = \Delta s - p_i(\Delta s + \Delta t) = \frac{\hat{\theta} - p_i}{1 - \hat{\theta}}\Delta t. \tag{14}$$

Note that the expected additional transfer decreases in belief $p_i$. Intuitively, the new program offers higher coverage and charges higher premium, which becomes less attractive to optimistic investors. Under both programs, a marginal investor with signal $x_i = \xi^*$ and belief $p_i = \hat{\theta}$ receives zero expected payoff and is indifferent between running or not. Therefore, the expected additional transfer, represented by Equation (14), is zero for the marginal investor. In equilibrium, all other participating investors receive better signals and form more optimistic beliefs than the marginal investor, i.e. $p_i > \hat{\theta}$. Therefore, Equation (14) is negative for all participating investors, and they expect to receive lower net transfer under the new program.

On the *extensive margin*, there are fewer participants under the new program unless both the old and the new programs are FPPs. This is intuitive because all participating investors with $p_i > \hat{\theta}$ are worse-off under the new program. Therefore, the most optimistic ones among them no longer find it optimal to participate. Mathematically, it is straightforward to verify that the participating threshold

$$\eta^*(s(\hat{\theta}, t), t) = \hat{\theta} + \sigma F^{-1}\left(\frac{\hat{\theta}t + 1 - \hat{\theta}R}{t + 1 - \hat{\theta}R}\right)$$

decreases in $t$. An exception is when both programs are FPPs. In this case, all staying investors, including the most optimistic ones, continue to participate, and the mass of participants with private signals $x_i \geq \xi^*$ remains the same.

In sum: the new program with higher tax charge incurs lower expected net transfer from the policymaker to investors because it attracts fewer participants and makes a lower expected transfer to each of them.

Recall from Proposition 3 that both PPPs and FPPs can achieve the same target fundamental threshold $\hat{\theta}$, however, PPPs charge higher tax $t$ than FPPs. Therefore, it follows directly from Proposition 4 that PPPs are less costly than FPPs to implement. We summarize this result in Corollary 1.

**Corollary 1** *Given any $\sigma > 0$ and any target fundamental threshold $\hat{\theta} \in [0, 1/R)$, any partial-participation program that implements $\hat{\theta}$ incurs lower implementation cost than any full-participation program that implements $\hat{\theta}$.*

In Section 2, we emphasized the key difference between PPPs and FPPs on the extensive margin: PPPs only attract pivotal investors with medium beliefs, while FPPs also attract the most optimistic investors. As we demonstrate here, on the intensive margin, every pivotal investor receives lower transfers in expectation under PPPs than under FPPs. Therefore, PPPs dominate FPPs in terms of implementation cost, and the optimal intervention program must be a PPP.

**Minimum cost** It follows directly from Proposition 4 that the least costly intervention program must be a PPP with the highest tax charge ($t \to R-1$). We summarize this result in Corollary 2.

**Corollary 2** *Given any $\sigma > 0$, for a partial-participation program that implements fundamental threshold $\hat{\theta} \in [0, 1/R]$, when $t \to R-1$, the implementation cost converges to its non-negative lower bound:*

$$C_{min}(\hat{\theta};\sigma) = \frac{\tau}{\overline{\theta}-\underline{\theta}} \int_{\hat{\theta}+\sigma F^{-1}(\hat{\theta})}^{\hat{\theta}+\sigma F^{-1}\left(\frac{1}{R}\right)} \left[1 - R \cdot F\left(\frac{x_i - \hat{\theta}}{\sigma}\right)\right] dx_i \geq 0. \tag{15}$$

The lower bound $C_{min}(\hat{\theta};\sigma)$ is simply the expected cost of implementing the least costly program – a PPP with tax charge $t$ converging to $R-1$. To see it clearly, recall from (13) that the belief of an investor with signal $x_i$ is $p_i = F\left(\frac{x_i-\hat{\theta}}{\sigma}\right)$. This investor expects to pay tax $t = R-1$ with probability $p_i$ and receive subsidy $s(\hat{\theta}, t) = 1$ with probability $1-p_i$.[14] Hence, the expected net transfer to this investor is $1 - Rp_i$, the integrand in (15). Moreover, as characterized in Proposition 2, the equilibrium run threshold and participation threshold are the two boundaries of the integration. Therefore, the integral represents the expected net transfer to all participating investors with signals $x_i \in [\hat{\theta}+\sigma F^{-1}(\hat{\theta}), \hat{\theta}+\sigma F^{-1}(1/R)]$ and corresponding beliefs $p_i \in [\hat{\theta}, 1/R]$. Multiplied by the welfare cost per unit of transfer $\tau$, we have the expected cost of implementation as in (15).

Thus far, we have determined the lower bound of the implementation cost when the policymaker offers a single intervention program. Is it possible to further reduce the cost of implementation with a menu of intervention programs? The answer is no, because the least costly program already minimizes the implementation cost on both intensive and extensive margins. To be more explicit, on the *intensive margin*, if an investor with belief $p_i$ participates in any other intervention program $(s,t)$ in equilibrium, her expected payoff (4) from doing so must be non-negative: $p_i(R-t-s) - (1-s) \geq 0$. Rearranging this inequality, we have

$$s(1-p_i) - tp_i \geq 1 - Rp_i,$$

which states that the expected net transfer to this investor (left-hand side) is always higher than that under the least costly program (right-hand side). On the *extensive margin*, the least costly program only attracts investors with beliefs $p_i \in [\hat{\theta}, 1/R]$, who would withdraw from the bank without the program. In contrast, as shown in Figure 3, any other intervention program $(s,t)$ – except for zero-participation programs – spends resources on some investors

[14] To be more precise, $t$ converges to $R-1$, and $s(\hat{\theta}, t)$ converges to 1.

with belief $p_i > 1/R$, who are optimistic enough to stay in the bank without intervention. Therefore, the least costly program incurs lower implementation cost than any other single or menu of several intervention programs, and $C_{min}(\hat{\theta}; \sigma)$ in (15) remains the lower bound of the implementation cost.

**Negligible information frictions** Lastly, we provide a proposition which characterizes the expected transfer in any intervention program in the limit of negligible information frictions ($\sigma \to 0$).

**Proposition 5** *For any intervention program $(\hat{\theta}, t)$ implementing $\hat{\theta} \in [0, 1/R)$, the expected transfer from the policymaker to the investors when $\sigma \to 0$ is given by*

$$\lim_{\sigma \to 0} \mathbb{E}_\theta \Gamma(\theta, \hat{\theta}, t; \sigma) = \begin{cases} -t\frac{\bar{\theta}-\hat{\theta}}{\bar{\theta}-\underline{\theta}} & \text{if } R - \frac{1}{\hat{\theta}} \leq t \leq 0, \\ 0 & \text{if } 0 < t < R - 1. \end{cases} \tag{16}$$

In the limit of negligible information frictions, both the investment threshold and the participation threshold in a partial-participation program converges to $\hat{\theta}$ (according to Proposition 2). This implies that the expected mass of participants in any PPP converges to zero for all realizations of $\theta$ *except* for the target threshold $\hat{\theta}$. Thus the expected cost of implementation converges to zero for all PPPs. In contrast, the mass of participants in any full-participation program is strictly positive when $\theta > \hat{\theta}$, and each participant receives a transfer of $-t \geq 0$ from the policymaker (or equivalently, pays a tax $t \leq 0$). So except for the government guarantee program with $t = 0$, all FPPs have a strictly positive expected cost of implementation. Although a government guarantee program and a partial-participation program both achieve zero expected cost, more investors participate in the former than in the latter. As a result, if investors' participation generates extra costs or creates distortions, a PPP will dominate a government guarantee program. For instance, to prevent bank runs with deposit insurance, policymakers need to set up deposit insurance funds. Even if no depositors file for claims, the administrative and opportunity costs of maintaining the fund can be enormous. Moreover, government guarantee programs can lead to moral hazard problems. In Section 4, we illustrate another advantage of PPPs: their robustness with respect to moral hazard.

# 4 Moral Hazard

Government guarantee programs are criticized for provoking moral hazard. In this section, we illustrate the second major advantage of partial-participation programs – robustness to moral hazard. Moral hazard can manifest itself in different forms under different contexts. In Section 4.1, we examine aggregate moral hazard problem at the bank level in the context of bank runs. In Section 4.2, we examine individual moral hazard problem at the investor level in the context of market freeze.

## 4.1 Aggregate Moral Hazard

It is widely accepted that the threat of bank runs can discipline moral hazard problems in financial institutions (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). For example, banks may shirk in credit assessment or make loans with a high default risk, and fund managers may engage in excessive risk taking because they do not bear the full cost when the downside of those risks becomes manifest. Investors can discipline financial institutions by the threat of early withdrawal. If a bank's portfolio is impaired, investors are likely to withdraw their funds early, which effectively increases the bank's incentive to maximize the performance of its investment portfolio. Although government guarantees (e.g., deposit insurance) deter panic-based runs, they reduce investors' skin in the game and therefore limit the disciplinary effect of runs. In this section, we show that PPPs are more efficient than FPPs – including government guarantee programs – in the presence of such moral hazard.

In order to incorporate moral hazard, we modify the benchmark model as follows. At $t = 0$, after the policymaker announces the intervention program but before the realization of $\theta$, the bank decides whether (or not) to exert effort in screening and monitoring its long-term investments. We use $i^e \in \{0, 1\}$ to denote the bank's effort choice. If the bank exerts effort ($i^e = 1$), its investments generate a high return and the bank will be able to pay the promised return of $R$ to the investors. However, if the bank shirks ($i^e = 0$), the quality of its investments deteriorate, and the bank will not be able to repay investors as promised. In this case, the return to the investors falls to $r \in (1, R)$, because of either a low recovery rate or the loss of convenience yield when the bank goes through a lengthy bankruptcy process.[15] In summary, an investor's payoff from staying and declining the policymaker's offered program ($a_i = 1_D$) is given by

$$\pi_D(\theta, l) = \begin{cases} Ri^e + r(1 - i^e) - 1 & \text{if } 1 - l \leq \theta, \\ -1 & \text{if } 1 - l > \theta; \end{cases}$$

[15] The reduction in return from $R$ to $r$ resembles the rents that a bank can extract with the threat to withdraw her loan collection skills in Diamond and Rajan (2001).

his payoff from staying and accepting the offer ($a_i = 1_A$) is given by

$$\pi_A(\theta, l) = \begin{cases} Ri^e + r(1 - i^e) - 1 - t & \text{if } 1 - l \leq \theta, \\ -1 + s & \text{if } 1 - l > \theta. \end{cases}$$

An investor's payoff from early withdrawal ($a_i = 0$) is normalized to 0. If the bank shirks, investors receive a lower payoff from staying in the bank, which gives them an incentive to monitor the bank. For simplicity, we assume that the bank's choice of effort $i^e$ is observable to investors so they can monitor the bank and condition their run decisions on the bank's effort choice.

The bank's payoff has two components. First, it incurs a cost $c^e > 0$ if it exerts effort but incurs no such cost if it shirks. Second, the bank receives a payoff $w > 0$ if it survives at $t = 2$ or a zero payoff if it defaults. We can therefore express the bank's payoff as

$$\pi^b(\theta, l) = -c^e i^e + w\mathbb{1}(1 - l \leq \theta).$$

In making its effort decision, the bank faces a trade-off. Shirking saves the cost of effort, but it also induces more runs by investors and so increases the probability of default. We impose the following assumption so that exerting effort ($i^e = 1$) is the socially optimal action.

**Assumption 1** *Shirking is inefficient,* $c^e < R - r$.

Hence the first-best scenario is one in which (a) the bank exerts effort and (b) all investors stay in the bank if and only if the fundamental $\theta \geq 0$. Everything else stays the same as in our benchmark model.

We showed in Section 3 that, if information frictions are nonnegligible ($\sigma > 0$), then PPPs incur strictly lower costs than do FPPs. Therefore, we shall next focus on the case of negligible information frictions ($\sigma \to 0$). In this case, government guarantee programs and PPPs both cost nothing to implement yet the latter are preferable because of their robustness to moral hazard.

The modified game with moral hazard can be solved via backward induction. Conditional on the bank's effort choice, investors follow a threshold investment strategy in equilibrium (as summarized in Proposition 2). Let $\theta_R = \frac{1-s}{R-t-s}$ be the fundamental threshold if the bank exerts effort, and let $\theta_r = \frac{1-s}{r-t-s}$ be the fundamental threshold if the bank shirks. In the limit of negligible information frictions, the probability of bank failure is proportional to the fundamental threshold ($\theta_R$ or $\theta_r$ depending on the bank's effort choice). Therefore, the bank is willing to exert effort if and only if the expected benefit from survival exceeds the cost of effort:

$$w\frac{\theta_r - \theta_R}{\bar{\theta} - \underline{\theta}} \geq c^e. \tag{17}$$

Since $\theta_r \leq 1/r$ and $\theta_R \geq 0$ under any intervention program, it follows that the expected benefit of effort is bounded from above by $w/r$. We make the following assumption about the cost of effort such that investors' monitoring is effective.

**Assumption 2** *Bank runs are sufficiently costly for the bank,* $c^e \leq \frac{w}{r(\bar{\theta}-\underline{\theta})}$.

If Assumption 2 is not valid, then the bank will always shirk regardless of any intervention program offered by the policymaker.

Recall from Proposition 2 that an intervention program with $t \leq 0$ is a FPP irrespective of the bank's effort choice. Because such programs provide free insurance against bank failures, they reduce investors' skin in the game and thus their incentive to monitor the bank. To be more explicit, consider a policymaker who seeks to achieve the first-best outcome with $i^e = 1$ and $\theta_R = 0$. In order to implement $\theta_R = 0$, a FPP must guarantee all losses due to bank failure (i.e., $s = 1$ and $t \leq 0$). Under such a program, however, investors have no incentive to monitor the bank. In other words: even if the bank shirks, investors' run decisions remain the same; that is, $\theta_r = \theta_R = 0$. So with a FPP, shirking is costless to the bank and so it will not exert effort ($i^e = 0$).

Now consider a partial-participation program that implements $\theta_R = 0$. By Proposition 2, this program will satisfy $s = 1$ and $0 < t \leq R - 1$. Although this PPP also guarantee all losses from bank failure, that guarantee is costly to the participating investors when the bank survives. When $t \leq r - 1$, the cost of participating in the program is so low that, even if the bank shirks, the PPP remains viable and implements the threshold $\theta_r = 0$. Hence the program is still too generous to induce investor monitoring, and the bank shirks because the probability of default does not change. However, if $r - 1 < t \leq R - 1$, then the cost of participating in the program is so high that, when the bank shirks, the PPP becomes a ZPP. Therefore, investors follow the laissez-faire threshold $\theta_r = 1/r$ if the bank shirks. From the bank's perspective, shirking increases the default probability from 0 to $1/r$. Assumption 2 implies that the bank would rather exert effort and thereby benefit from the intervention program's stabilizing effect.

We summarize the foregoing analysis in our next proposition.[16]

**Proposition 6** *When the information frictions are negligible* ($\sigma \to 0$), (a) *no FPP achieves the first-best outcome and* (b) *any PPP* $(s, t)$ *with* $s = 1$ *and* $r - 1 < t < R - 1$ *does achieve the first-best outcome.*

[16] The proof of the proposition follows directly from the foregoing analysis.

Under a FPP, the policymaker faces a trade-off between coordination efficiency and effort efficiency. On the one hand, insuring investors against losses from bank failure incentivizes them to stay in the bank. On the other hand, that guarantee reduces their incentive to monitor the bank. If the policymaker aims to eliminate coordination failure, then she must tolerate the bank's resulting moral hazard problem. In contrast, a PPP not only deters bank runs but also prevents banks from shirking. The subsidy incentivizes investors to stay in the bank while the tax restores their incentive to monitor the bank. So if a bank shirks then the tax discourages investors from participating in the program, which eliminates its stabilizing effect and thereby increases the likelihood of bank runs. Hence the threat of bank runs prevents banks from shirking. Therefore, a PPP with a well-designed tax-to-subsidy ratio preserves investors' disciplinary role in reducing the bank's moral hazard.

## 4.2 Individual Moral Hazard

Although we have focused on bank runs so far, our model has a wide range of applications. In this section, we apply our model to study *market freeze* due to coordination failure. Moral hazard in this context is likely to emerge at individual level. We then show that partial-participation programs are robust to individual moral hazard as well.

It is widely agreed that coordination failures among financial institutions and investors contributed to the 2008 financial crisis.[17] Various asset markets suffered from runs and liquidity dry-up, and asset prices fell well below their fundamentals.[18] The credit market also froze, because banks hoard liquidity when anticipating a sluggish real economy.[19] For illustration, we focus on credit market freeze and reinterpret our model. Unlike the benchmark model, the strategic players are now *banks* – instead of investors – who individually decide whether to lend to the real economy ($a_i$). If enough banks lend ($a_i = 1$), there is sufficient aggregate credit to support the growth of real economy, and each bank profits from its lending. However, if enough banks abstain from lending ($a_i = 0$), the resulting credit contraction slows down economy growth and generates a loss for banks who lend. This gives rise to a so-called self-fulling credit freeze, during which banks withdraw credit supply en

[17]See Brunnermeier (2009) and Gorton (2010) for a review of the factors, which are reminiscent of bank runs, that contributed to that crisis.

[18]Among others, Gorton and Metrick (2012) and Covitz, Liang and Suarez (2013) document investor runs on the market for repos and asset-backed commercial paper. Bernardo and Welch (2004) model runs on financial markets in which investors pre-emptively sell assets because they fear a surge in future liquidation costs. Brunnermeier and Pedersen (2009) present a model of the feedback between asset market liquidity and traders' funding liquidity, which can result in a sudden dry-up of liquidity and in asset prices moving away from their fundamentals.

[19]Bebchuk and Goldstein (2011) model self-fulfilling credit market freezes when banks abstain from lending because they expect other banks to reduce their supply of credit contributing to a sluggish real economy.

masse (Bebchuk and Goldstein, 2011).

Government guarantees reduce banks' losses from lending and encourage credit supply. However, they also reduce banks' skin in the game and their incentive to screen and monitor borrowers. Recall that in the context of bank runs, moral hazard problem arises at the aggregate level: if a bank shirks, all of its investors suffer from a lower payoff. In contrast, here, moral hazard emerges at the individual level: if a bank shirks in screening and monitoring, it only reduces the bank's own profitability and does not directly spill over to other banks.

To incorporate individual moral hazard, we extend our benchmark model into a two-stage game. The first stage is the same as the benchmark model *except* that banks do not receive their payoffs until the second stage. If the realized fundamental $\theta < 1 - l$ in the first stage, then the credit market freezes and all lending banks suffer a net loss normalized to 1. If the first-stage realized fundamental $\theta \geq 1-l$, then aggregate credit supply is sufficient, and each lending bank's payoff depends on its individual effort choice in the second stage. If a bank exerts effort to screen and monitor its borrowers, it pays a cost of effort $c^e > 0$ and earns a profit $R - 1$ with probability 1. Although shirking saves the cost of effort, it reduces banks' profitability. Specifically, if a lending bank shirks, it will suffer a loss of 1 with probability $\gamma \geq 0$ and will earn a profit $R - 1$ with probability $1 - \gamma$.

As in the benchmark model, we consider intervention programs characterized by a subsidy-tax combination $(s, t)$. An important friction here is that the policymaker can *not* contract on the effort choice of each individual bank; instead, the tax and subsidy payments can be contingent only on the payoff of each individual bank. In light of this friction, an intervention program stipulates that a participating bank must pay tax $t$ if it profits from lending and will receive a subsidy $s$ if it suffers a loss due to either credit market freeze or shirking. We make the following assumptions about the parameters involved.

**Assumption 3** *Banks' payoff has the following properties:*

*(i) shirking is inefficient,* $c^e < \gamma R$;

*(ii) lending is ex ante efficient,* $c^e < R - 1$.

This assumption implies that the first-best scenario is that (a) all banks lend and exert effort if the fundamental $\theta \geq 0$ and (b) all banks abstain from lending otherwise. Assumption 3 also implies that, if aggregate credit supply is sufficient ($\theta \geq 1-l$), a nonparticipating bank will exert effort for sure; however, a participating bank exerts effort if and only if its incentive compatibility (IC) constraint,

$$R - t - c^e \geq (1-\gamma)(R-t) + \gamma s, \tag{18}$$

is met. Therefore, to induce effort from participating banks, there is an upper bound on the sum of tax and subsidy:

$$t + s \leq R - \frac{c^e}{\gamma}. \tag{19}$$

We remark that $t + s$ represents the intervention program's distortion of the difference in the payoff from exerting effort versus shirking. If $t + s$ exceeds the threshold, then banks do not have enough skin in the game to exert effort, resulting in shirking. With a higher cost of effort $c^e$ or with a lower probability $\gamma$ of losses due to shirking, the incentive problem is more severe and so imposes a tighter constraint on the intervention program's generosity. We therefore make the following assumption to focus on the parameter region in which the IC constraint is binding.

**Assumption 4** *The cost of effort is high,* $c^e > \gamma(R - 1)$.

It will become evident later that, in this parameter region, the policymaker faces a trade-off between inducing effort and reducing coordination failure. As in Section 4.1, we focus on the limit of negligible information friction. In this case, government guarantee programs and PPPs both cost nothing to implement yet the latter are preferable because of their robustness to individual moral hazard.

**Government Guarantee** As in the benchmark model, all banks will participate in a government guarantee program with $t = 0$ and $s > 0$. The expected payoff for a bank with interim belief $p_i$ is therefore

$$\mathbb{E}[\pi_A(\theta, l) \mid p_i] = \begin{cases} p_i(R - c^e) + (1 - p_i)s - 1 & \text{if } \; s \leq R - \frac{c^e}{\gamma}, \\ p_i(1 - \gamma)R + (1 - p_i + p_i\gamma)s - 1 & \text{if } \; s > R - \frac{c^e}{\gamma}. \end{cases} \tag{20}$$

Our analysis of the benchmark model established that, in the unique equilibrium, the fundamental threshold below which credit market freezes equals the belief of the marginal bank who is indifferent between lending or not. If the policymaker prevents shirking by setting $s \leq R - c^e/\gamma$, then in equilibrium the fundamental threshold becomes

$$\theta^* = \frac{1 - s}{R - c^e - s}. \tag{21}$$

However, Assumption 4 implies that the most ambitious program with $s = R - c^e/\gamma < 1$ cannot insure against all losses from lending. Therefore, programs aimed to prevent shirking cannot achieve the first-best outcome with $\theta^* = 0$.

If the policymaker tolerates shirking by setting $s > R - c^e/\gamma$, then the fundamental threshold becomes

$$\theta^* = \frac{1-s}{(1-\gamma)(R-s)}. \tag{22}$$

Indeed, the policymaker can eliminate all coordination failure and achieve $\theta^* = 0$ with a program with $s = 1$ that insures against any loss from lending. Yet in that case, the policymaker must tolerate the inefficiency due to the resulting moral hazard. In summary, within the framework of a government guarantee program, the policymaker faces a trade-off between inducing effort and encouraging lending. Next we show that, by implementing a partial-participation program, the policymaker can both encourage lending and induce effort at the same time.

**Partial-Participation Programs** Now we look for a partial-participation program that can implement the first-best outcome. As in the main model, ensuring the effectiveness of a PPP requires that we set the tax and subsidy such that (a) the most optimistic banks choose $a_i = 1_D$ and (b) the pivotal banks, with interim beliefs around $1/R$, choose $a_i = 1_A$.

Banks who opt out of the program ($a_i = 1_D$) endogenize all the profits or losses from their lending. By Assumption 3(i), they always exert effort. In the presence of moral hazard, the option to shirk renders the intervention program more attractive. An optimistic bank, who believes that the aggregate credit supply is sufficient for sure ($p_i = 1$), will take action $a_i = 1_D$ if and only if

$$R - c^e - 1 > \max\{R - c^e - 1 - t, (1-\gamma)(R-t) + \gamma s - 1\}. \tag{23}$$

According to this inequality, a bank's payoff from $a_i = 1_D$ is higher than that from $a_i = 1_A$ regardless of whether it exert efforts or shirks. Hence condition (23) ensures that the option to shirk does not make the intervention program *too* attractive, and the most optimistic banks choose $a_i = 1_D$.

A bank with interim belief $p_i = 1/R$ is indifferent between $a_i = 1_D$ and $a_i = 0$, since both generate zero payoff. Under an intervention program $(s, t)$, the bank strictly prefers $a_i = 1_A$ if and only if its payoff from doing so is positive:

$$\max\left\{\frac{1}{R}(R - c^e - t) + \left(1 - \frac{1}{R}\right)s - 1, \frac{1}{R}(1-\gamma)(R-t) + \left(1 - \frac{1}{R} + \frac{1}{R}\gamma\right)s - 1\right\} > 0. \tag{24}$$

This inequality guarantees that banks with interim beliefs $p_i = 1/R$ *strictly* prefers $a_i = 1_A$, and therefore, banks with beliefs slightly below $1/R$ will choose $a_i = 1_A$ regardless of effort choice. Absent an intervention program, they would abstain from lending ($a_i = 0$). Yet the

PPP gives them more incentive to lend, which boosts aggregate credit supply. All investors are aware of the increase in aggregate credit supply and so become more optimistic about overall economic prospects and their return from lending. Therefore, the mere announcement of PPP can effectively boost the supply of credit to the real economy.

This extension echoes our main model in the sense that, as information friction $\sigma$ approaches zero, so does the mass of banks who participate in the program. Recall that banks who opt out of the program always exert effort. So even if participating banks shirk, the inefficiency is limited by their small mass. In contrast to government guarantee programs, partial-participation programs can achieve both goals: minimizing coordination failure and inducing effort.

**Proposition 7** *Given Assumptions 3 and 4, if $\sigma \to 0$, then no government guarantee program can restore the first-best outcome. In contrast, the equilibrium outcome under a partial-participation program $(s, t)$ with $s = 1$ and $\frac{c^e - \gamma R + \gamma}{1-\gamma} < t < R - 1$ converges to the first-best outcome. Furthermore, the ex ante cost of implementing such a partial-participation program also converges to zero.*

This proposition demonstrates the advantage of partial-participation programs over government guarantee programs when moral hazard problem is severe. The subsidy of an intervention program encourages lending in the first stage but deters effort in the second stage. Under a government guarantee program, the policymaker faces the trade-off between first-stage lending efficiency and second-stage effort efficiency. Yet notwithstanding the moral hazard problem, a PPP still achieves the first-best outcome at zero cost. The advantage of PPPs with respect to individual moral hazard is that they involve only a small mass of participating banks. Although these banks shirk in the second stage, the effect of that shirking on social welfare is limited because the mass of these banks approaches zero as the information friction vanishes. In general: PPPs are preferable to FPPs, such as government guarantees, in the presence of inefficiencies that are proportional to the mass of participants.

# 5 Implementation

Our model provides novel insights for policy design while being agnostic about the actual forms of implementation. In this section, we discuss the practical aspects of implementing partial-participation programs to reduce coordination failure. Directly following the model, we first consider a policymaker as the provider of the intervention program in section 5.1. We then examine the viability of privatizing the intervention program in section 5.2.

## 5.1 Policymaker-Sponsored Intervention Programs

In the context of panic-driven bank runs as in our main model,[20] a partial-participation program (PPP) maps to an insurance program with two features distinct from regular deposit insurance. First, participation is *voluntary* at the *depositor* level. Second, the price of the insurance induces partial participation.

In terms of implementation scheme, the most straightforward interpretation is that a policymaker or a policymaker-sponsored entity directly offers such an insurance program to depositors. The low cost of implementation implies that the policymaker can downsize the deposit insurance fund compared with the current one to back a regular deposit insurance. Alternatively, a policymaker can decentralize deposit insurance by subsidizing banks who offer such insurance programs to their depositors. To enhance their credibility as the insurer, banks can hold safe and liquid assets – reminiscent of deposit insurance fund – to back the insurance. Compared with the centralized implementation scheme, a potential drawback is that individual banks are exposed to bank-specific shocks and therefore need to hold more safe and liquid assets than a centralized policymaker does to back the insurance program.

As we discussed in Section 4.2, another example of coordination failures in the financial system is *market freeze*. During the great financial crisis, various asset and credit markets suffered from runs and liquidity dry-up. To support liquidity in asset and credit markets, governments around the world provided substantial assistance – including granting access to cheap funding and acquiring illiquid assets – to financial institutions. This imposed a heavy fiscal burden and induced moral hazard problems.[21] In this context, to take advantage of partial-participation programs, policymakers should make the financial assistance unattractive to those financial institutions that are optimistic about the recovery and capable of obtaining finance on their own.

In fact, the Capital Purchase Program (CPP) in the Troubled Asset Relief Program (TARP) adopted by the US Treasury can be viewed as an example of a partial-participation program. Under CPP, eligible banks could receive capital injection from the Treasury in the form of preferred stock or debt securities. In return, participating banks were required to issue warrants to the Treasury so the Treasury could purchase common shares or other securities in the future. The requirement to issue warrants forced participating banks to

[20] The concept of a "bank" can be generalized to any financial institution susceptible to run risk, including corporate bond mutual funds (Chen, Goldstein and Jiang, 2010). Correspondingly, we can incorporate generalized payoff structure of investors in these financial institutions as demonstrated in Appendix B.

[21] Over the period 2008–2014, accumulated gross financial sector assistance amounted to 8% of the eurozone's gross domestic product (Domingues Semeano, Ferdinandusse et al., 2018). In some European countries, government rescues of financial institutions during the financial crisis ended up contributing to the subsequent sovereign debt crisis (Acharya, Drechsler and Schnabl, 2014).

share the upside of future recovery with the government and provided strong incentives for optimistic and healthy banks to opt out. Bayazitova and Shivdasani (2012) provide evidence of banks self-selecting into CPP: strong banks opted out and weak banks participated.

## 5.2 Privatized Intervention Programs

Previously, we have been considering policymaker-sponsored insurance programs that induce partial participation. In this section, we discuss the viability of the private sector as the insurance provider without subsidies from the policymaker.

We first consider a private insurer as the provider of such an insurance program. The viability relies on whether a private insurer can profit from doing so. Within our model, the answer is no, because as shown in Corollary 2, the expected cost of implementing a PPP is always non-negative. However, a private insurer may find it profitable for reasons outside the model. For example, in the context of bank runs, a bank can benefit from reducing costly default induced by panic runs. Therefore, the bank may have the incentive to self-run the insurance program without subsidies from the policymaker. More broadly, if investors are more risk-averse than a private insurer, there may be gains from trade for the private insurer to provide such an insurance program.

Although a private insurer may have incentive to provide the insurance program, we outline two caveats associated with privatized programs. First, the investors may face counterparty risk, because the insurer has to make substantial payments when the fundamentals fall slightly below the fundamental threshold. Second, our proposed partial-participation program is a *ex ante* program, which is independent of the realized fundamentals. This requires commitment power from the insurer. A profit-maximizing private insurer has strong incentive to acquire information about the fundamentals and adjust the terms of the insurance program ($s$ and $t$) accordingly. The public information conveyed by the terms of the insurance program can cause additional fragility and undermine the efficiency of the insurance program (Angeletos, Hellwig and Pavan, 2006).

Next, we consider the viability of a market-based insurance program, such as the trading of credit default swap (CDS) among investors. To illustrate the idea, consider a self-fulfilling debt crisis, in which investors lose confidence in a government and refuse to purchase government debt. This inhibits the government from rolling over its debt, leading to a self-fulfilling default (Cole and Kehoe, 2000). In this context, sovereign CDS – a costly insurance by definition – is similar to our proposed partial-participation programs. Specifically, pessimistic investors can purchase CDS to reduce the risk associated with government debt, while optimistic investors can directly invest in government debt or even sell CDS protections. This can encourage more investors to purchase government debt and relieve concerns about coordination failures. However, the two caveats about a single private insurer carry over. First, investors may get concerned about counterparty risk, which is likely to elevate during crises. This hinders protection provided by CDS and its role in alleviating coordination failure among investors. Second, the price of CDS can serve as a public signal about the aggregate opinion of investors. In particular, a high CDS price reveals overall pessimism about the solvency of the government and discourages investors from investing in government debt (Shen, 2021).

# 6 Conclusions

This study analyzes panic-based runs resulting from coordination failures and proposes a novel type of intervention program that policymakers can utilize to deter runs. The proposed intervention program screens and supports pivotal investors who receive medium signals. Due to strategic complementarities and amplification by higher-order beliefs, correctly incentivizing these pivotal investors has a significant effect on all investors. Compared with conventional government guarantee programs, the partial-participation programs proposed in this paper is not only less costly to implement but also more robust to moral hazard.

Our proposed partial-participation program has a wide range of applications in preventing coordination failures in financial systems. Here we remark its limitations in certain economic environments. First, a PPP requires the policymaker to observe investors' actions and then to condition her offer of the program on their run decisions. This may impose additional costs on the policymaker to verify investors' actions. Second, the effectiveness of our proposed program relies on investors being rational. If the rationality of investors is bounded, then the amplification effect (via higher-order beliefs) will accordingly be limited.

## References


**Acharya, Viral**, **Itamar Drechsler** and **Philipp Schnabl**, "A Pyrrhic Victory? Bank Bailouts and Sovereign Credit Risk", *The Journal of Finance*, 2014, *69* (6), 2689–2739.

**Allen, Franklin**, **Elena Carletti**, **Itay Goldstein** and **Agnese Leonello**, "Moral Hazard and Government Guarantees in the Banking Industry", *Journal of Financial Regulation*, 2015, *1* (1), 30–50.

**Allen, Franklin**, **Elena Carletti**, **Itay Goldstein** and **Agnese Leonello**, "Government Guarantees and Financial Stability", *Journal of Economic Theory*, 2018, *177*, 518–557.

**Angeletos, George-Marios**, **Christian Hellwig** and **Alessandro Pavan**, "Signaling in a Global Game: Coordination and Policy Traps", *Journal of Political Economy*, 2006, *114* (3), 452–484.

**Artavanis, Nikolaos**, **Daniel Paravisini**, **Claudia Robles-Garcia**, **Amit Seru** and **Margarita Tsoutsoura**, "Deposit Withdrawals", *Unpublished working paper*, 2019.

**Basak, Deepal** and **Zhen Zhou**, "Diffusing Coordination Risk", *American Economic Review*, 2020, *110* (1), 271–97.

**Bayazitova, Dinara** and **Anil Shivdasani**, "Assessing TARP", *Review of Financial Studies*, 2012, *25* (2), 377–407.

**Bebchuk, Lucian A** and **Itay Goldstein**, "Self-Fulfilling Credit Market Freezes", *Review of Financial Studies*, 2011, *24* (11), 3519–3555.

**Bernardo, Antonio E** and **Ivo Welch**, "Liquidity and Financial Market Runs", *The Quarterly Journal of Economics*, 2004, *119* (1), 135–158.

**Bernstein, Shai** and **Eyal Winter**, "Contracting with Heterogeneous Externalities", *American Economic Journal: Microeconomics*, 2012, *4* (2), 50–76.

**Bond, Philip** and **Kathleen Hagerty**, "Preventing Crime Waves", *American Economic Journal: Microeconomics*, 2010, *2* (3), 138–59.

**Bond, Philip** and **Rohini Pande**, "Coordinating Development: Can Income-Based Incentive Schemes Eliminate Pareto Inferior Equilibria?", *Journal of Development Economics*, 2007, *83* (2), 368–391.

**Brunnermeier, Markus K**, "Deciphering The Liquidity and Credit Crunch 2007-2008", *Journal of Economic Perspectives*, 2009, *23*(1), 77–100.

**Brunnermeier, Markus K** and **Lasse Heje Pedersen**, "Market Liquidity and Funding Liquidity", *The Review of Financial Studies*, 2009, *22*(6), 2201–2238.

**Calomiris, Charles W** and **Charles M Kahn**, "The Role of Demandable Debt in Structuring Optimal Banking Arrangements", *The American Economic Review*, 1991, pages 497–513.

**Carlsson, Hans** and **Eric Van Damme**, "Global Games and Equilibrium Selection", *Econometrica: Journal of the Econometric Society*, 1993, pages 989–1018.

**Chen, Qi**, **Itay Goldstein** and **Wei Jiang**, "Payoff Complementarities and Financial Fragility: Evidence From Mutual Fund Outflows", *Journal of Financial Economics*, 2010, *97*(2), 239–262.

**Choi, Dong Beom**, "Heterogeneity and Stability: Bolster the Strong, Not the Weak", *Review of Financial Studies*, 2014, *27*(6), 1830–1867.

**Cole, Harold L** and **Timothy J Kehoe**, "Self-fulfilling debt crises", *The Review of Economic Studies*, 2000, *67*(1), 91–116.

**Cong, Lin William**, **Steven R Grenadier** and **Yunzhi Hu**, "Dynamic Interventions and Informational Linkages", *Journal of Financial Economics*, 2020, *135*(1), 1–15.

**Cooper, Russell** and **Thomas W Ross**, "Bank Runs: Deposit Insurance and Capital Requirements", *International Economic Review*, 2002, *43*(1), 55–72.

**Covitz, Daniel**, **Nellie Liang** and **Gustavo A Suarez**, "The Evolution of A Financial Crisis: Collapse of The Asset-Backed Commercial Paper Market", *The Journal of Finance*, 2013, *68*(3), 815–848.

**Demirgüç-Kunt, Asli** and **Enrica Detragiache**, "Does Deposit Insurance Increase Banking System Stability? An Empirical Investigation", *Journal of monetary economics*, 2002, *49*(7), 1373–1406.

**Demirgüç-Kunt, Asli** and **Harry Huizinga**, "Market Discipline and Deposit Insurance", *Journal of monetary economics*, 2004, *51*(2), 375–399.

**Diamond, Douglas W** and **Philip H Dybvig**, "Bank Runs, Deposit Insurance, and Liquidity", *Journal of political economy*, 1983, *91*(3), 401–419.

**Diamond, Douglas W** and **Raghuram G Rajan**, "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking", *Journal of Political Economy*, 2001, *109*(2), 287–327.

**Domingues Semeano, João**, **Marien Ferdinandusse et al.**, "The Fiscal Impact of Financial Sector Support Measures: Where Do We Stand A Decade On From The Financial Crisis?", *Economic Bulletin Boxes*, 2018, *6*.

**Farhi, Emmanuel** and **Jean Tirole**, "Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops", *The Review of Economic Studies*, 2018, *85*(3), 1781–1823.

**Frankel, David M**, **Stephen Morris** and **Ady Pauzner**, "Equilibrium Selection In Global Games With Strategic Complementarities", *Journal of Economic Theory*, 2003, *108*(1), 1–44.

**Goldstein, Itay** and **Ady Pauzner**, "Demand-Deposit Contracts and the Probability of Bank Runs", *the Journal of Finance*, 2005, *60*(3), 1293–1327.

**Gorton, Gary** and **Andrew Metrick**, "Securitized Banking and The Run on Repo", *Journal of Financial Economics*, 2012, *104*(3), 425–451.

**Gorton, Gary B**, *Slapped By The Invisible Hand: The Panic of 2007*, Oxford University Press 2010.

**He, Zhiguo** and **Wei Xiong**, "Dynamic Debt Runs", *Review of Financial Studies*, 2012, *25*(6), 1799–1843.

**Ioannidou, Vasso P** and **María Fabiana Penas**, "Deposit Insurance and Bank Risk-Taking: Evidence From Internal Loan Ratings", *Journal of Financial Intermediation*, 2010, *19*(1), 95–115.

**Leonello, Agnese**, "Government Guarantees and the Two-Way Feedback Between Banking and Sovereign Debt Crises", *Journal of Financial Economics*, 2018, *130*(3), 592–619.

**Morris, Stephen** and **Mehdi Shadmehr**, "Reward and Punishment in a Regime Change Game", Technical report, Unpublished, 2017.

**Morris, Stephen** and **Hyun Song Shin**, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks", *American Economic Review*, 1998, pages 587–597.

**Morris, Stephen** and **Hyun Song Shin**, "Global Games: Theory and Applications", *Advances in Economics and Econometrics*, 2003, page 56.

**Morris, Stephen** and **Hyun Song Shin**, "Coordination Risk and the Price of Debt", *European Economic Review*, 2004, *48* (1), 133–153.

**Rochet, Jean-Charles** and **Xavier Vives**, "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?", *Journal of the European Economic Association*, 2004, *2* (6), 1116–1147.

**Sakovics, Jozsef** and **Jakub Steiner**, "Who Matters in Coordination Problems?", *The American Economic Review*, 2012, *102* (7), 3439–3461.

**Segal, Ilya**, "Coordination and Discrimination in Contracting with Externalities: Divide and Conquer?", *Journal of Economic Theory*, 2003, *113* (2), 147–181.

**Shen, Lin**, "Capital Flows in the Financial System and Supply of Credit", *Available at SSRN 3847983*, 2021.

# Appendices

## A Optimal Choice of Target Threshold

In this section we investigate the determinants of the optimal choice of $\hat{\theta}$ when there is no moral hazard problem. First we take the partial derivative of $C_{min}$ with respect to $\hat{\theta}$:

$$\frac{\mathrm{d}}{\mathrm{d}\hat{\theta}} C_{min}(\hat{\theta};\sigma) = -\frac{\sigma\tau}{\bar{\theta}-\underline{\theta}} \frac{1-R\hat{\theta}}{f(F^{-1}(\hat{\theta}))} > 0. \tag{A.1}$$

Next we write down the total expected payoff of the investors by taking expectation of (7),

$$\mathbb{E}_{\theta}[W(\theta,\hat{\theta};\sigma)] = \frac{R}{\bar{\theta}-\underline{\theta}} \int_{\hat{\theta}}^{\bar{\theta}} \left[1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta - \frac{1}{\bar{\theta}-\underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left[1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta, \tag{A.2}$$

where $\xi^* = \hat{\theta} + \sigma F^{-1}(\hat{\theta})$. Taking derivative with respect to $\hat{\theta}$, we have

$$\frac{\mathrm{d}}{\mathrm{d}\hat{\theta}} \mathbb{E}_{\theta}[W(\theta,\hat{\theta};\sigma)] = -\frac{R(1-\hat{\theta})}{\bar{\theta}-\underline{\theta}} - \frac{\mathrm{d}\xi^*}{\mathrm{d}\hat{\theta}} \frac{1}{\sigma(\bar{\theta}-\underline{\theta})} \left[R \int_{\hat{\theta}}^{\bar{\theta}} f\left(\frac{\xi^* - \theta}{\sigma}\right) \mathrm{d}\theta - \int_{\underline{\theta}}^{\bar{\theta}} f\left(\frac{\xi^* - \theta}{\sigma}\right) \mathrm{d}\theta\right], \tag{A.3}$$

$$= -\frac{R(1-\hat{\theta})}{\bar{\theta}-\underline{\theta}} - \frac{\mathrm{d}\xi^*}{\mathrm{d}\hat{\theta}} \frac{R\hat{\theta}-1}{\bar{\theta}-\underline{\theta}}, \tag{A.4}$$

$$= -\frac{R-1}{\bar{\theta}-\underline{\theta}} + \frac{\sigma(1-R\hat{\theta})}{(\bar{\theta}-\underline{\theta}) f(F^{-1}(\hat{\theta}))}. \tag{A.5}$$

When $\hat{\theta} \in [0, 1/R]$, the derivative of the total social welfare w.r.t. $\hat{\theta}$ is

$$\frac{\mathrm{d}}{\mathrm{d}\hat{\theta}} \mathbb{E}_{\theta}[W(\theta,\hat{\theta};\sigma)] - \frac{\mathrm{d}}{\mathrm{d}\hat{\theta}} C_{min}(\hat{\theta};\sigma) = -\frac{R-1}{\bar{\theta}-\underline{\theta}} + (1+\tau) \frac{\sigma(1-R\hat{\theta})}{(\bar{\theta}-\underline{\theta}) f(F^{-1}(\hat{\theta}))}. \tag{A.6}$$

Although the full characterization of the optimal fundamental threshold $\hat{\theta}$ requires additional assumptions on the distribution of noises $F(\varepsilon)$, we are able to provide some comparative statics on the relationship between the optimal threshold and exogenous parameters. Notice that the right hand side of (A.6) is increasing in $\tau$ and $\sigma$ while it is decreasing in $R$. The following comparative statics follow immediately from Topkis's theorem. First, notice that when there exist positive information frictions ($\sigma > 0$), the optimal $\hat{\theta}$ can be greater than zero – the first best fundamental threshold when there is no information friction. This is because information frictions can lead to imperfect coordination: a positive mass of investors stay in the bank when $\theta$ falls below the fundamental threshold. When $\hat{\theta}$ decreases, the run threshold $\xi^*$ becomes relatively smaller compared to $\hat{\theta}$, and this makes the imperfect coordination problem more severe and more costly for the policy maker. Therefore, the optimal $\hat{\theta}$ must increase with $\sigma$. Second, a higher social cost of transfer $\tau$ implies a higher cost of imperfect coordination, as the policymaker needs to compensate the losses of the investors who suffer bank failure. Hence, the optimal $\hat{\theta}$ must increase with $\tau$. Third, a higher gross return $R$ implies a larger benefit of encouraging investment. In addition, a higher gross return $R$ gives investors stronger incentives to invest outside of the optimal partial-participation program; it reduces the mass of participants and lowers the cost of imperfect coordination. Since both effects work in the same direction, the optimal $\hat{\theta}$ decreases with $R$.

In summary, with non-negligible information friction ($\sigma > 0$), the socially optimal fundamental threshold $\hat{\theta}$ increases with the size of information friction $\sigma$, increases with the welfare cost $\tau$ per unit of transfer, and decreases with the long-term return $R$. In the limit of vanishing information friction ($\sigma \to 0$), the optimal $\hat{\theta}$ converges to the first-best fundamental threshold $\theta^{FB} = 0$.

# B General Payoff Structure

In the main model, we focus on regime-switching payoff structure and show that PPPs dominate FPPs in terms of implementation cost. In this appendix, we illustrate the robustness of the result to continuous payoff structure. In particular, we follow the set-ups of the symmetric binary-action global games given by Morris and Shin (2003) and allow for general monotonic payoff functions. In addition, at the end of this appendix, we discuss informally the role of uniform prior distribution and provide a sufficient condition such that the result holds under non-uniform prior distributions.

**Payoff Structure** As in the benchmark model presented in Section 2, an investor's payoff from withdrawing early ($a_i = 0$) is normalized to 0. However, an investor's payoff from staying ($a_i = 1$) is now modified to be a function $\pi(x_i, l)$, which is weakly increasing in the private signal $x_i$ and the total amount of funding that stays in the bank $l = \int_0^1 a_i \, \mathrm{d}i$.[22] The fundamental $\theta$ is distributed uniformly on $[\underline{\theta}, \bar{\theta}]$, where $\underline{\theta} < -\sigma$ and $\bar{\theta} > 1 + \sigma$. The private signal received by investor $i$ is $x_i = \theta + \sigma\varepsilon_i$, where the $\varepsilon_i$ are i.i.d. with density function $f(\varepsilon)$ and distribution function $F(\varepsilon)$ with support $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

[22] To simplify the demonstration, we assume that the payoff is a function of the private signal, not of the fundamentals; our results continue to hold under the alternative payoff structure $\pi(\theta, l)$. See Morris and Shin (2003) for the discussion of the two approaches.

We consider only linear intervention programs, since they are realistic and easy to implement. In principle, we could also consider nonlinear intervention programs; yet as we shall demonstrate, linear intervention programs already deliver the results in the benchmark model. An intervention program $(s,t)$ consists of two parts: a lump-sum subsidy $s \geq 0$ and a proportional tax $t \in [0,1]$.[23] An investor who accepts the offer receives the lump-sum subsidy $s$ and pays the proportional tax $t$ once the coordination outcome is realized. His payoff from accepting the offer is[24]

$$\pi_A(x_i, l) = (1-t)\pi_D(x_i, l) + s, \tag{B.1}$$

where $\pi_D(x_i, l) = \pi(x_i, l)$ is his payoff from declining the offer. Investors who receive low private signals anticipate a low realization of the fundamental $\theta$ and a low funding condition $l$, so they are pessimistic about their payoffs from staying in the bank. Hence they expect that the tax they pay will be low and so are more willing (than are optimistic investors) to accept the offer. Recall from our main model that partial-participation programs do not appeal to the most optimistic agents, which reduces the implementation costs and reduces moral hazard. The proportional tax $t$ reflects this feature and helps target agents who receive medium signals.

We adopt several of the literature's standard assumptions regarding the payoff function.

**Assumption B.1** *The payoff function $\pi(x_i, l)$ satisfies the following properties.*

*(i) (Monotonicity) The payoff function $\pi(x_i, l)$ is weakly increasing in $x_i$ and weakly increasing in $l$.*

*(ii) (Strict Laplacian State Monotonicity) $\int_0^1 \pi(x_i, l)\,\mathrm{d}l$ is strictly increasing in $x_i$.*

*(iii) (Limit Dominance) There exist $\theta_0, \theta_1 \in \left(\underline{\theta} + \frac{1}{2}\sigma, \bar{\theta} - \frac{1}{2}\sigma\right)$ such that*

$$\pi(x_i, 1) < 0 \quad \textit{for all } x_i < \theta_0, \tag{B.2}$$

$$\pi(x_i, 0) > 0 \quad \textit{for all } x_i > \theta_1. \tag{B.3}$$

*(iv) (Continuity) $\int_0^1 g(l)\pi(x_i, l)\,\mathrm{d}l$ is continuous in $x_i$ and density function $g$.*

Part (i) of Assumption B.1 gives the strategic complementarities among investors: an investor's payoff from staying in the bank increases when the total funding staying the

[23] Programs with $s < 0$ correspond to zero-participation programs in the benchmark model.

[24] One might notice that, when $\pi(x, l) < 0$, investors end up paying a "negative tax". Let $\underline{\pi} = \pi\left(\underline{\theta} - \frac{1}{2}\sigma, 0\right)$ be the lower bound of the payoff. Then the intervention program can be implemented by providing a positive subsidy $s - t\underline{\pi}$ and imposing a proportional tax $t$ on the positive tax base $\pi(x, l) - \underline{\pi}$.

bank $l$ increases or when his private signal $x_i$ increases indicating a higher fundamental $\theta$. Note that the payoff function need not be strictly increasing or continuous. For example, the payoff function in our benchmark model in Section 2 is a step function. The role of part (ii) is to ensure that the equilibrium is unique when it exists, with or without the intervention program. Part (iii) ensures the existence of two dominance regions – so that we can undertake the iterated deletion of dominated strategies from both sides. Part (iv) governs the integration of the payoff function so that there always exists an equilibrium.

**Equilibrium Characterization** The equilibrium without intervention is characterized in the following proposition. The coordination outcome in this case serves as a benchmark to highlight the effect of intervention programs.

**Proposition B.1** *Without intervention* ($s = t = 0$)*, there is a unique equilibrium whereby each investor stays in the bank if and only if his private signal* $x_i \geq \xi_0^*$. *The signal threshold* $\xi_0^*$ *is given by*

$$\int_0^1 \pi(\xi_0^*, l)\,\mathrm{d}l = 0.$$

Let us compare the coordination results characterized in Proposition B.1 with the first-best outcome. The first-best scenario is that all investors follow the same cut-off strategy $\theta_0$, which is the upper bound for the lower dominance region. By Assumption B.1, the coordination outcome $\xi_0^* > \theta_0$ unless $\pi(\theta_0, l) = 0$ for any $l \in [0, 1]$. In other words: if the realized fundamental $\theta \in (\theta_0, \xi_0^*)$, then a panic-based bank run will arise. Hence the goal of intervention is to reduce the coordination threshold from $\xi_0^*$ to a value that is as close to $\theta_0$ as possible.

Next we analyze the equilibrium with an intervention program $(s, t)$.

**Definition B.1** *A program* $(s, t)$ *is an intervention program with target* $\xi^* \in (\theta_0, \xi_0^*)$ *if and only if it satisfies the following two conditions:*

*(i) (Intervention Target)* $s/(1-t) = -\int_0^1 \pi(\xi^*, l)\,\mathrm{d}l$;

*(ii) (Lower Dominance Region)* $s/(1-t) < -\pi(\underline{\theta}, 1)$.

Denoting by $G(\sigma; s, t)$ the coordination game with information friction $\sigma$ and intervention program $(s, t)$, we can characterize the Bayesian Nash equilibria with intervention programs.

**Proposition B.2** *Given an intervention program* $(s, t)$ *with target* $\xi^* \in (\theta_0, \xi_0^*)$*, there exists a unique Bayesian Nash equilibrium of the coordination game* $G(\sigma; s, t)$.

1. *If* $s/t > \pi(\bar{x}, 1)$*, the program is a full-participation program, in which*

$$a_i(x_i) = \begin{cases} 1_A & \text{if } x_i \geq \xi^*, \\ 0 & \text{if } x_i < \xi^*. \end{cases}$$

2. *If* $s/t \leq \pi(\bar{x}, 1)$*, the program is a partial-participation program, in which*

$$a_i(x_i) = \begin{cases} 1_D & \text{if } x_i \geq \eta^*(\sigma), \\ 1_A & \text{if } \xi^* \leq x_i < \eta^*(\sigma), \\ 0 & \text{if } x_i < \xi^*. \end{cases}$$

*Moreover, if* $s/t < \pi(\xi^*, 1)$*,* $\eta^*(\sigma)$ *converges to* $\xi^*$ *as* $\sigma$ *goes to 0.*

Proposition B.3 characterizes the sets of FPPs and PPPs that reduce the run threshold to a given $\xi^*$.

**Proposition B.3** *If* $\int_0^1 \pi(\theta_0, l)\,\mathrm{d}l \geq \pi(\underline{\theta}, 1)$*, then for any* $\xi^* \in (\theta_0, \xi_0^*)$*, there exist a non-empty set of full-participation programs* $\left\{(s,t) \mid t \in (0, t^*), s = -(1-t)\int_0^1 \pi(\xi^*, l)\,\mathrm{d}l\right\}$ *and a non-empty set of partial-participation programs* $\left\{(s,t) \mid t \in [t^*, 1), s = -(1-t)\int_0^1 \pi(\xi^*, l)\,\mathrm{d}l\right\}$ *with target* $\xi^*$*.*

In the presence of an intervention program, all investors become more optimistic about their investment payoff. It follows that the lower dominance region, where agents prefer to run on the bank even if $l = 1$, shrinks. The condition stipulated in Proposition B.3 guarantees that the lower dominance region still exists even when there is an intervention program. If that condition is violated, then there could be multiple equilibria when targeting $\xi^*$ close to $\theta_0$. However, if we follow the equilibrium refinements proposed in Goldstein and Pauzner (2005), then we can select the equilibrium described in Proposition B.2 even without the lower dominance region. So following those refinements, there always exists a partial-participation program that restores the first-best scenario. Moreover, the lower dominance region may disappear because we limit our attention to programs with linear transfers. Linear transfer schedules typically pay out high subsidies when the fundamentals are low. If the policymaker reduces subsidies in the case of extremely low realizations of the fundamentals (i.e., when $\theta < \theta_0$) or if she increases the convexity of the tax schedule appropriately, then both the left dominance region and the equilibrium's uniqueness can be recovered. In any case, there always exists an intervention program that can restore the first-best scenario.

**Expected transfers** In Section 3 we show that conditional on the same target fundamental threshold, PPPs have lower expected net transfer from the policy maker to participating investors in the regime switching game than FPPs. This result still holds in the general coordination game, as demonstrated in the following proposition.

**Proposition B.4** *Conditional on a target fundamental threshold $\xi^*$, the expected transfer from the policy maker to investors decreases in $t$.*

Proposition B.4 is the counterpart of Proposition 4 in the general model. The proof and the intuition of these two propositions are very similar. In particular, Proposition B.4 implies that conditional on the same target threshold, any PPP has lower expected transfer than any FPP.

**Non-uniform prior distribution** In both the benchmark model and the general model, we have assumed a uniform prior distribution of the fundamentals. It is well known that the uniqueness of equilibrium in global games depends on the distribution of the fundamentals. When the prior distribution of the fundamentals contains relative precise information with respect to the private signals, the coordination game may have multiple equilibria.[25] With multiple equilibria, the welfare implication of different intervention programs becomes ambiguous. However, as long as the uniqueness of equilibrium is maintained, our proposed PPPs require less expected transfer than FPPs when targeting at the same threshold.

To generalize our result to non-uniform prior, it suffices to impose an additional assumption on the expected payoffs of the threshold investors.

**Assumption B.2** *The expected payoff of a threshold investor*

$$U_D(k,k) = \int_{\underline{\theta}}^{\bar{\theta}} p(\theta|x=k)\pi\left(k, 1 - F\left(\frac{k-\theta}{\sigma}\right)\right) \mathrm{d}\theta$$

*strictly increases in $k$.*

Under Assumption B.2, any linear intervention program induces a unique threshold equilibrium. Proposition B.1-B.3 still hold with slight modifications to the notations in the proof. Moreover, the welfare implication of PPPs and FPPs in Proposition B.4 is unchanged since the proof does not rely on the prior distribution of the fundamentals.

[25] See Morris and Shin (2003) for a reference.

# C Ex Ante Heterogeneity

Sakovics and Steiner (2012) (referred to as SS hereafter) analyzes the optimal subsidy programs in coordination games with ex ante heterogeneity. In particular, they argue that cost-effective subsidy programs should target influential and insensitive agents with certain ex ante characteristics. By contrast, our paper highlights the advantages of screening agents based on their interim beliefs rather than ex ante characteristics. In this appendix, we demonstrate the robustness of our main result to ex ante heterogeneity and compare the intuitions with those in SS.

We first generalize our model to incorporate ex ante heterogeneity à la SS. Suppose now there are $G$ groups of heterogeneous investors, indexed by $g \in \{1, ..., G\}$, each group with mass $m_g$. The investors are heterogeneous in three aspects: weight $w_g$, return $R_g$ and noise distribution $F_g$. Specifically, the aggregate early withdrawal in the payoff function (Equation 1) is given by $1 - l = \sum_{g=1}^{G} \int_0^{m_g} w_g a_i^g di$, where $a_i^g$ is the action of investor $i$ in group $g$. The weight $w_g$ can be interpreted as the size of deposit held by an investor in group $g$, and it represents the investor's influence on the coordination outcome. We normalize $\sum_{g=1}^{G} w_g m_g = 1$ such that the aggregate early withdrawal is between 0 and 1. Moreover, investors differ in their deposit returns $R_g$. Lastly, the distributions of signal noise are group-specific. An investor $i$ in group g receives a private noise $x_i^g = \theta + \sigma\varepsilon_i^g$, where $\varepsilon_i^g$ follows a distribution $F_g(\cdot)$ with a support $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

We consider subsidy-tax intervention programs $(s, t)$ as in the main model. Importantly, the intervention program $(s, t)$ is *not* group-specific, and consequently, the policymaker does not need to observe the ex ante characteristics of individual investors. In contrast, SS considers group-specific subsidy programs, which requires the policymaker to observe ex ante characteristics of individual investors. In what follows, we focus on partial-participation programs and compare them with group-specific subsidy programs in SS. Let $R_{min} = \min\{R_1, ..., R_g\}$. Proposition C.1 below generalizes Proposition 2 in the main model and characterizes the equilibrium with partial-participation programs.

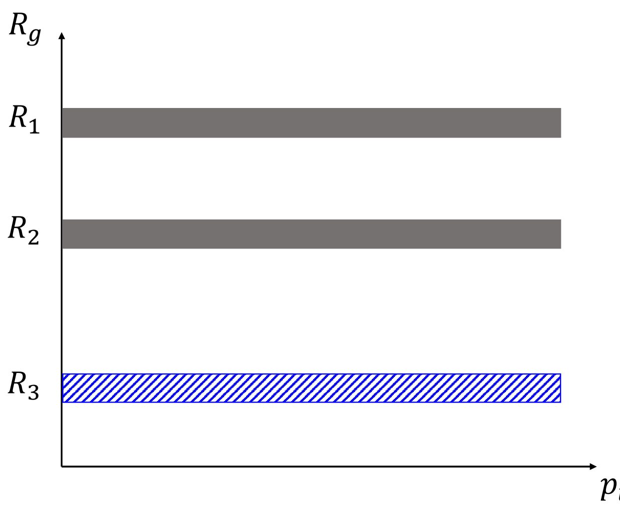
**Proposition C.1** *When the policymaker offers a subsidy-tax program $(s, t)$ with $s \in [0, 1]$ and $0 < t < s \cdot (R_{min} - 1)$, the game has a unique equilibrium in which investor i in group g follows a threshold strategy:*

$$a_i^g(x_i^g) = \begin{cases} 1_D & \text{if } x_i^g \geq \eta_g^*(s,t), \\ 1_A & \text{if } \xi_g^*(s,t) \leq x_i^g < \eta_g^*(s,t), \\ 0 & \text{if } x_i^g < \xi_g^*(s,t); \end{cases}$$

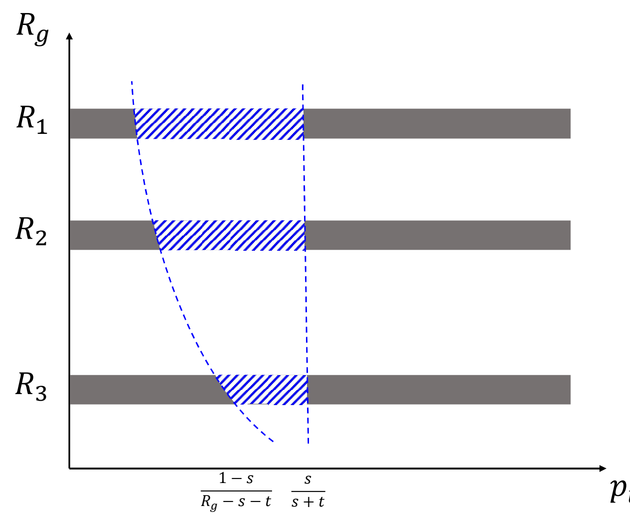
*where*

$$\xi_g^*(s,t) = \sum_{g=1}^{G} m_g w_g \frac{1-s}{R_g - s - t} + \sigma F_g^{-1}\left(\frac{1-s}{R_g - s - t}\right),$$
$$\eta_g^*(s,t) = \sum_{g=1}^{G} m_g w_g \frac{1-s}{R_g - s - t} + \sigma F_g^{-1}\left(\frac{s}{s+t}\right).$$

Same as in the main model, PPPs effectively screen investors based on their interim beliefs and identify a small mass of "pivotal" investors with medium beliefs. In contrast, Proposition 2 in SS suggests that subsidy programs should target ex ante "pivotal" investors with the highest $w_g/R_g$ ratio, i.e. investors who are the most influential and the least sensitive to the coordination result. Figure 6 visually compares the "pivotal" investors in SS (Panel a) and in this paper (Panel b). For illustration, we consider three groups of investors characterized by their distinct return $R_g$ (vertical axis). Within each group, investors receive different private signals and form heterogeneous interim beliefs (horizontal axis).



(a) Type-Specific Subsidy Programs (SS)



(b) Partial-Participation Programs

Figure 6: "Pivotal" Investors

The comparison between Panel (a) and (b) yields the main difference between the two types of programs. Type-specific subsidy programs (Panel a) target at the third group of investors with the lowest $R_g$ irrespective of their interim beliefs. In other words, subsidy programs in SS map to FPPs in this paper for the third group of investors. On the contrary, PPPs (Panel b) screen for marginal investors with medium beliefs in *all* groups. Therefore, the two main advantages of PPPs over FPPs, cost efficiency and robustness to moral hazard, carry over. First, PPPs make use of resources more efficiently and incur lower implementation cost than subsidy programs in SS. This is because SS subsidizes optimistic investors who would stay in the bank even without the subsidy and pessimistic investors who would run on the bank even with the subsidy. In contrast, PPPs identify investors who are at the margin of running. On the one hand, the marginal investors are the least costly to persuade to stay. On the other hand, amplified by higher-order beliefs, their action to stay has a cascade effect on deterring panic-based runs as illustrated in Figure 4. In fact, in the limiting case of $\sigma \to 0$, PPPs can achieve the first-best outcome, as summarized in Proposition C.2 below. Second, PPPs preserve investors "skin in the game" and therefore are more robust to moral hazard problems than subsidy programs in SS.

**Proposition C.2** *When the information frictions are negligible ($\sigma \to 0$), the equilibrium outcome under a partial-participation program $(s, t)$ with $s = 1$ and $1 < t < R_{min} - 1$ converges to the first-best outcome. Furthermore, the expected cost of implementing such a partial-participation program converges to zero.*

In summary, partial-participation programs are robust to ex ante heterogeneity. In equilibrium, marginal investors of all types self-select to participate, and their participation boosts all investors' confidence and effectively deters panic-based runs. The comparison between PPPs and subsidy programs in SS highlights that policymakers should screen for *interim* "pivotal" investors rather than target at *ex ante* "pivotal" investors. Doing so not only saves cost of implementation but also enhances resilience to moral hazard problems. Last but not least, such screening does not require policymakers to observe investors' ex ante characteristics and therefore could save potential cost of information acquisition.

# D Proofs

**Proof of Proposition 1.** It follows directly the proof of Proposition 2.1 in Morris and Shin (2003) using iterated deletion of dominated strategies. ■

**Proof of Proposition 2.** Frankel, Morris and Pauzner (2003) prove existence, uniqueness, and monotonicity in multi-action global games. Since our set-up does not satisfy the continuity assumption, we provide our own proof here.

In case 1, $a_i = 1_D$ is dominated by $a_i = 1_A$ and so all investors who choose to stay will accept the offer. We can therefore set the payoff from staying at $\pi(\theta, l) = \pi_A(\theta, l)$ and directly apply Proposition 1. In case 3, $a_i = 1_A$ is similarly dominated by $a_i = 1_D$ and so the offer is never accepted by any investor. Hence the equilibrium is exactly the same as in Proposition 1.

In case 2, the optimal strategy of investor $i$ conditional on his interim belief $p_i$ is

$$a_i = \begin{cases} 1_D & \text{if } p_i > p_2^*, \\ 1_A & \text{if } p_1^* < p_i \leq p_2^*, \\ 0 & \text{if } p_i \leq p_1^*; \end{cases}$$

here $p_1^* = \frac{1-s}{R-t-s}$ and $p_2^* = \frac{s}{t+s}$ are two threshold beliefs that satisfy $0 \leq p_1^* < p_2^* \leq 1$. Note that the coordination outcome (i.e., the bank's survival) depends only on the aggregate early withdrawal $1-l$; whether investors accept or decline the offer has no direct effect on the bank's survival. In other words, the fundamental threshold $\theta^*(\xi)$ defined in (2) is a function of run threshold $\xi$ only. Therefore, we can focus on investors' run decision and define a sequence $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$, starting with $(\underline{\xi}_0, \bar{\xi}_0) = (-\infty, +\infty)$, such that a strategy survives $n$ rounds of iterated deletion of dominated strategies if and only if

$$a_i = 0 \quad \text{if } x_i < \underline{\xi}_n, \tag{D.1}$$

$$a_i \in \{1_D, 1_A\} \quad \text{if } x_i \geq \bar{\xi}_n, \tag{D.2}$$

where the recursive expression for $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ is

$$\underline{\xi}_{n+1} = \inf\{x : p(x; \underline{\xi}_n) \geq p_1^*\}, \tag{D.3}$$

$$\bar{\xi}_{n+1} = \sup\{x : p(x; \bar{\xi}_n) \leq p_1^*\}. \tag{D.4}$$

Applying the same techniques used in the proof of Proposition 1, we can prove that the limit of the two threshold sequences both converge to

$$\xi^*(s,t) = p_1^* + \sigma F^{-1}(p_1^*), \tag{D.5}$$

which is the run threshold in the unique Bayesian Nash equilibrium of the global game. The associated participation threshold $\eta$ is the solution to

$$p(\eta; \xi^*(s,t)) = p_2^*. \tag{D.6}$$

Solving (D.6) yields

$$\eta^*(s,t) = p_1^* + \sigma F^{-1}(p_2^*). \tag{D.7}$$

Plugging in the expressions for the two threshold beliefs $p_1^* = \frac{1-s}{R-t-s}$ and $p_2^* = \frac{s}{t+s}$ completes the proof. ■

**Proof of Proposition 3.** As we show in Section 2, ZPPs can only implement the the "lassiez-faire" threshold $\hat{\theta} = 1/R$. Therefore, if an intervention program $(s,t)$ implements $\hat{\theta} \in [0, 1/R)$, the program must be a FPP or a PPP. In any case, the fundamental threshold achieved by $(s,t)$ can be expressed as:

$$\theta^*(\xi^*(s,t)) = \frac{1-s}{R-t-s}.$$

Equating $\theta^*(\xi^*(s,t)) = \hat{\theta}$, it is straightforward to show that the intervention program $(s,t)$ must satisfy

$$s = \frac{\hat{\theta}}{1-\hat{\theta}}t + \frac{1-\hat{\theta}R}{1-\hat{\theta}}. \tag{D.8}$$

Notice that $s \geq 0$ requires $t \geq R - \frac{1}{\hat{\theta}}$. According to Proposition 2, if $(s,t)$ is a FPP, $t \leq 0$; if $(s,t)$ is a PPP, $0 < t < s(R-1)$. Substituting in Equation (D.8), if $(s,t)$ is a PPP, $0 < t < R-1$.

It is also straightforward to verify that any $(s,t)$ satisfying $R - \frac{1}{\hat{\theta}} \leq t < R-1$ and Equation (D.8) indeed implements $\hat{\theta}$. Therefore, the set of FPPs that implement $\hat{\theta}$ is

$$\left\{(s,t) : R - \frac{1}{\hat{\theta}} \leq t \leq 0, s = \frac{\hat{\theta}}{1-\hat{\theta}}t + \frac{1-\hat{\theta}R}{1-\hat{\theta}}\right\},$$

and the set of PPPs that implement $\hat{\theta}$ is

$$\left\{(s,t) : 0 < t < R-1, s = \frac{\hat{\theta}}{1-\hat{\theta}}t + \frac{1-\hat{\theta}R}{1-\hat{\theta}}\right\}.$$

Since $\hat{\theta} < \frac{1}{R}$ and $R > 1$, both sets are non-empty. ■

**Proof of Proposition 4.** The expected net transfer of intervention program $(\hat{\theta}, t)$ can be expressed as

$$\begin{aligned}\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)] = \frac{s(\hat{\theta},t)}{\bar{\theta}-\underline{\theta}} \int_{\underline{\theta}}^{\hat{\theta}} \left[F\left(\frac{\eta^*(\hat{\theta},t)-\theta}{\sigma}\right) - F\left(\frac{\xi^*(\hat{\theta})-\theta}{\sigma}\right)\right] d\theta \\ - \frac{t}{\bar{\theta}-\underline{\theta}} \int_{\hat{\theta}}^{\bar{\theta}} \left[F\left(\frac{\eta^*(\hat{\theta},t)-\theta}{\sigma}\right) - F\left(\frac{\xi^*(\hat{\theta})-\theta}{\sigma}\right)\right] d\theta,\end{aligned}$$

where $s(\hat{\theta},t)=\frac{\hat{\theta}t+1-\hat{\theta}R}{1-\hat{\theta}}$, $\xi^*(\hat{\theta})=\hat{\theta}+\sigma F^{-1}\left(\hat{\theta}\right)$ and

$$\eta^*(\hat{\theta},t)=\begin{cases}\bar{\theta}+\frac{\sigma}{2} & \text{if } t\leq 0,\\ \hat{\theta}+\sigma F^{-1}\left(\frac{\hat{\theta}t+1-\hat{\theta}R}{t+1-\hat{\theta}R}\right) & \text{if } 0<t<R-1.\end{cases}$$

Note that when $t\leq 0$, the program is a FPP, and the participation threshold $\eta^*(\hat{\theta},t)$ is the upper bound of signal realization.

**Continuity** It is evident that expected net transfer $\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)]$ is continuous at any $t\neq 0$. Below we verify its continuity at $t=0$. Note that $\lim_{t\to 0^+}\eta^*(\hat{\theta},t)=\hat{\theta}+\frac{\sigma}{2}$. Therefore,

$$\lim_{t\to 0^+}\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)]=\frac{s(\hat{\theta},t)}{\bar{\theta}-\underline{\theta}}\int_{\underline{\theta}}^{\hat{\theta}}\left[1-F\left(\frac{\xi^*-\theta}{\sigma}\right)\right]d\theta=\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},0;\sigma)],$$

and expected net transfer $\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)]$ is also continuous at any $t$.

**Monotonicity** Given any $\hat{\theta}\in\left[0,\frac{1}{R}\right)$, to simplify the notation, we shall no longer indicate the dependence of $\xi^*$ and $\eta^*$ on $\hat{\theta}$ in what follows. Observe that

$$\int_\alpha^\beta\left[F\left(\frac{\eta^*-\theta}{\sigma}\right)-F\left(\frac{\xi^*-\theta}{\sigma}\right)\right]\mathrm{d}\theta=\int_{\xi^*}^{\eta^*}\left[F\left(\frac{x-\alpha}{\sigma}\right)-F\left(\frac{x-\beta}{\sigma}\right)\right]\mathrm{d}x.$$

We can re-express the expected net transfer as follows:

$$\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)]=\begin{cases}\frac{1}{\bar{\theta}-\underline{\theta}}\int_{\xi^*}^{\bar{\theta}+\frac{1}{2}\sigma}\left[\frac{\hat{\theta}t+1-\hat{\theta}R}{1-\hat{\theta}}-\frac{t+1-\hat{\theta}R}{1-\hat{\theta}}F\left(\frac{x-\hat{\theta}}{\sigma}\right)+tF\left(\frac{x-\bar{\theta}}{\sigma}\right)\right]dx & \text{if } R-\frac{1}{\hat{\theta}}\leq t\leq 0,\\ \frac{1}{\bar{\theta}-\underline{\theta}}\int_{\xi^*}^{\eta^*(t)}\left[\frac{\hat{\theta}t+1-\hat{\theta}R}{1-\hat{\theta}}-\frac{t+1-\hat{\theta}R}{1-\hat{\theta}}F\left(\frac{x-\hat{\theta}}{\sigma}\right)\right]dx & \text{if } 0<t<R-1.\end{cases} \tag{D.9}$$

Take the derivative with respect to $t$. When $t\leq 0$,

$$\begin{aligned}\frac{d\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)]}{dt}&=\frac{1}{\bar{\theta}-\underline{\theta}}\left(\int_{\xi^*}^{\bar{\theta}-\frac{1}{2}\sigma}\left[\frac{\hat{\theta}}{1-\hat{\theta}}-\frac{1}{1-\hat{\theta}}F\left(\frac{x-\hat{\theta}}{\sigma}\right)\right]dx+\int_{\bar{\theta}-\frac{1}{2}\sigma}^{\bar{\theta}+\frac{1}{2}\sigma}\left[F\left(\frac{x-\bar{\theta}}{\sigma}\right)-1\right]dx\right)\\&<\frac{1}{\bar{\theta}-\underline{\theta}}\int_{\xi^*}^{\bar{\theta}-\frac{1}{2}\sigma}\left[\frac{\hat{\theta}}{1-\hat{\theta}}-\frac{1}{1-\hat{\theta}}F\left(\frac{\xi^*-\hat{\theta}}{\sigma}\right)\right]dx<0;\end{aligned}$$

when $0 < t < R-1$,

$$\begin{aligned}\frac{d\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)]}{dt} &= \frac{1}{\bar{\theta}-\underline{\theta}}\int_{\xi^*}^{\eta^*(t)}\left[\frac{\hat{\theta}}{1-\hat{\theta}}-\frac{1}{1-\hat{\theta}}F\left(\frac{x-\hat{\theta}}{\sigma}\right)\right]dx \\ &< \frac{1}{\bar{\theta}-\underline{\theta}}\int_{\xi^*}^{\eta^*(t)}\left[\frac{\hat{\theta}}{1-\hat{\theta}}-\frac{1}{1-\hat{\theta}}F\left(\frac{\xi^*-\hat{\theta}}{\sigma}\right)\right]dx = 0.\end{aligned}$$

Therefore, $\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)]$ strictly decreases in $t \in [R-\frac{1}{\hat{\theta}}, R-1)$. ■

**Proof of Corollary 1.** Given any partial-participation program $(\hat{\theta}, t^P)$ and any full-participation program $(\hat{\theta}, t^F)$ that achieve the same fundamental threshold $\hat{\theta}$, we have $t^F \leq 0 < t^P$. Since $\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)]$ strictly decreases in $t$, we have

$$\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t^P;\sigma)] < \mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t^F;\sigma)].$$

■

**Proof of Corollary 2.** Proposition 4 implies that the intervention program with the lowest net transfer to implement $\hat{\theta}$ must be a PPP with $t \to R-1$. Take the limit of Equation (D.9):

$$\lim_{t\to R-1}\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)] = \frac{1}{\bar{\theta}-\underline{\theta}}\int_{\hat{\theta}+\sigma F^{-1}(\hat{\theta})}^{\hat{\theta}+\sigma F^{-1}\left(\frac{1}{R}\right)}\left[1-R\cdot F\left(\frac{x-\hat{\theta}}{\sigma}\right)\right]dx.$$

Therefore, the lower bound for cost of implementing $\hat{\theta}$ is

$$C_{min}(\hat{\theta};\sigma) = \tau\lim_{t\to R-1}\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)] = \frac{\tau}{\bar{\theta}-\underline{\theta}}\int_{\hat{\theta}+\sigma F^{-1}(\hat{\theta})}^{\hat{\theta}+\sigma F^{-1}\left(\frac{1}{R}\right)}\left[1-R\cdot F\left(\frac{x-\hat{\theta}}{\sigma}\right)\right]dx.$$

It is straightforward to verify that $C_{min}(\hat{\theta};\sigma) \geq 0$ for any $\hat{\theta} \in [0, 1/R]$ and $\sigma > 0$. ■

**Proof of Proposition 5.** We prove by taking the limit of (D.9) when $\sigma \to 0$.

If $R - \frac{1}{\hat{\theta}} \leq t \leq 0$, the program is a full-participation program. In the limit, the run threshold converges to the fundamental threshold: $\lim_{\sigma\to 0}\xi^* = \hat{\theta}$. For any $x > \hat{\theta}$, $\lim_{\sigma\to 0} F\left(\frac{x-\hat{\theta}}{\sigma}\right) = 1$. For any $x < \bar{\theta}$, $\lim_{\sigma\to 0} F\left(\frac{x-\bar{\theta}}{\sigma}\right) = 0$. Thus

$$\lim_{\sigma\to 0}\mathbb{E}_\theta[\Gamma(\theta,\hat{\theta},t;\sigma)] = \lim_{\sigma\to 0}\frac{1}{\bar{\theta}-\underline{\theta}}\int_{\hat{\theta}}^{\bar{\theta}}\left[\frac{\hat{\theta}t+1-\hat{\theta}R}{1-\hat{\theta}}-\frac{t+1-\hat{\theta}R}{1-\hat{\theta}}\right]dx = -t\frac{\bar{\theta}-\hat{\theta}}{\bar{\theta}-\underline{\theta}} \geq 0.$$

If $0 < t < R-1$, the program is a partial-participation program. In the limit, both run threshold and participation threshold converges to the fundamental threshold: $\lim_{\sigma\to 0} \xi^* = \lim_{\sigma\to 0} \eta^*(t) = \hat{\theta}$. Since the boundaries of integration both converges to $\hat{\theta}$, and the integrand is bounded, it follows naturally that $\lim_{\sigma\to 0} \mathbb{E}_\theta[\Gamma(\theta, \hat{\theta}, t; \sigma)] = 0$. ■

**Proof of Proposition 6.** A proof has been provided in Section 4. ■

**Proof of Proposition 7.** The government guarantee program that achieves a target fundamental threshold $\hat{\theta} = 0$ is $(s,t) = (1,0)$, which guarantees any loss from failed investment. However, such program violates the incentive compatibility constraint (condition (18)) and so all participating investors would shirk, incurring welfare loss.

Next we show that a partial-participation program $(s,t)$ with $s = 1$ and $t \in \big(\frac{c^e - \gamma R + \gamma}{1-\gamma}, R - 1\big)$ can restore the first-best outcome when $\sigma \to 0$. Under such program, investor $i$'s optimal strategy – conditional on his interim belief – is

$$a_i = \begin{cases} 1_D & \text{if } p_i \geq p_2^*, \\ 1_A & \text{if } p_1^* \leq p_i < p_2^*, \\ 0 & \text{if } p_i < p_1^*; \end{cases}$$

here

$$p_2^* = \frac{s}{\gamma R + (t+s)(1-\gamma) - c^e} < 1,$$
$$p_1^* = \frac{1-s}{(1-\gamma)(R-t-s)} = 0.$$

Following the proof of Proposition 2(ii), we can establish the existence of a unique equilibrium in which any investor $i$ adopts the following strategy:

$$a_i = \begin{cases} 1_D & \text{if } x_i \geq \eta^*, \\ 1_A & \text{if } \xi^* \leq x_i < \eta^*, \\ 0 & \text{if } x_i < \xi^*; \end{cases}$$

here

$$\xi^* = p_1^* + \sigma F^{-1}(p_1^*),$$
$$\eta^* = p_1^* + \sigma F^{-1}(p_2^*).$$

When $\sigma \to 0$, both $\xi^*$ and $\eta^*$ converge to $p_1^* = 0$. Hence this PPP achieves the first-best outcome: for any fundamental $\theta > 0$, all investors take action $a_i = 1_D$ and exert effort; for any fundamental $\theta < 0$, all investors take action $a_i = 0$. The mass of participants in the program is zero (except when $\theta = 0$); hence for any continuous distribution of the fundamental, the program's ex ante cost of implementation converges to zero. ■

**Proof of Proposition B.1.** Consider an investor who receives a private signal $x_i$ and knows that all other investors follow threshold strategy $k$ – that is, run on the bank if and only if the private signal $x_i < k$. Then the expected payoff from staying in the bank is

$$U(k, x_i) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x_i - \theta}{\sigma}\right) \pi\left(x_i, 1 - F\left(\frac{k - \theta}{\sigma}\right)\right) \mathrm{d}\theta. \tag{D.10}$$

Note that $U(k, x_i)$ weakly decreases with $k$ and weakly increases with $x_i$. That is: an investor has higher expected payoff from staying in the bank if (a) others are more willing to stay in the bank or (b) the investor receives a high signal indicating a high fundamental $\theta$.

Next we prove the uniqueness of equilibrium via iterated deletion of dominated strategies. The strategy profile of an investor amounts to his action as a function of his private signal. We denote this profile by $a_i(x_i)\colon \mathbb{R} \to \{0, 1\}$. We will prove that a strategy survives $n$ rounds of iterated deletion of dominated strategies if and only if

$$a_i(x_i) = \begin{cases} 0 & \text{if } x_i < \underline{\xi}_n, \\ 1 & \text{if } x_i \geq \bar{\xi}_n. \end{cases} \tag{D.11}$$

Here $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ satisfies

$$-\infty = \underline{\xi}_0 < \underline{\xi}_1 \leq \cdots \leq \underline{\xi}_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \tag{D.12}$$

This result can be proved by induction. Let the starting nodes be $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = +\infty$ so that there are no restrictions on investors' strategies. Consider all strategy profiles that survive $n \in \mathbb{N}$ rounds of deletion. In round $n+1$, the most *optimistic* belief for an investor is that all other investors follow a threshold strategy $\underline{\xi}_n$. So for any $x_i$ such that $U(\underline{\xi}_n, x_i) < 0$, we have that staying ($a_i(x_i) = 1$) is strictly dominated by early withdrawing ($a_i(x_i) = 0$). The most *pessimistic* belief for an investor is, analogously, that all other investors follow a threshold strategy $\bar{\xi}_n$. So for $x_i$ such that $U(\bar{\xi}_n, x_i) > 0$, any strategy profile with $a_i(x_i) = 0$ is strictly dominated by $a_i(x_i) = 1$.

Because $U(k, x_i)$ is non-decreasing in $x_i$, a strategy profile that survives deletion of dominated strategies must satisfy the restrictions given in (D.11) with $(\underline{\xi}_{n+1}, \bar{\xi}_{n+1})$ defined inductively as

$$\underline{\xi}_{n+1} = \inf\{x : U(\underline{\xi}_n, x_i) \geq 0\}, \tag{D.13}$$

$$\bar{\xi}_{n+1} = \sup\{x : U(\bar{\xi}_n, x_i) \leq 0\}. \tag{D.14}$$

The monotonicity of $U(k, x_i)$ guarantees that $\underline{\xi}_{n+1} \leq \bar{\xi}_{n+1}$. Note that the Limit Dominance assumption (part (iii) of Assumption B.1) implies $U(-\infty, x_i) < 0$ for $x_i < \theta_0$ and $U(+\infty, x_i) > 0$ for $x_i > \theta_1$, and so $\underline{\xi}_1 > -\infty$ and $\bar{\xi}_1 < +\infty$. Hence $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ is a well-defined sequence of real couples that satisfy (D.12).

We have proved that each of $\{\underline{\xi}_n\}_{n=1}^{\infty}$ and $\{\bar{\xi}_n\}_{n=1}^{\infty}$ is a monotonic and bounded sequence. Hence they converge to (respectively) the finite numbers $\underline{\xi}$ and $\bar{\xi}$ when $n \to \infty$. The definitions (D.13) and (D.14) imply that $U(\underline{\xi}, \underline{\xi}) \geq 0$ and $U(\bar{\xi}, \bar{\xi}) \leq 0$.

For any $y \in [\theta_0, \theta_1]$, we have that

$$U(y, y) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{y - \theta}{\sigma}\right) \pi\left(y, F\left(\frac{y - \theta}{\sigma}\right)\right) \mathrm{d}\theta = \int_0^1 \pi(y, l)\, \mathrm{d}l,$$

which strictly increases with $y$ by the Strict Laplacian State Monotonicity (part (ii) of Assumption B.1). Therefore, $U(\underline{\xi}, \underline{\xi}) \geq 0 \geq U(\bar{\xi}, \bar{\xi})$ implies that $\underline{\xi} \geq \bar{\xi}$. Recall the fact that $\underline{\xi}_{n+1} \leq \bar{\xi}_{n+1}$, which implies $\underline{\xi} \leq \bar{\xi}$. Hence it must be the case that $\underline{\xi} = \bar{\xi} = \xi_0^*$ where $y = \xi_0^*$ is the unique solution to $U(y, y) = 0$. Therefore, the only strategy that survives the iterated deletion of dominated strategies is the threshold strategy $\xi_0^*$. ■

**Proof of Proposition B.2.** Consider an investor who receives a private signal $x_i$ and believes that all other investors follow threshold strategy $k$. His expected payoff from staying in the bank and declining the intervention offer is

$$U_D(k, x_i) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x_i - \theta}{\sigma}\right) \pi\left(x_i, 1 - F\left(\frac{k - \theta}{\sigma}\right)\right) \mathrm{d}\theta.$$

The expected payoff from staying in the bank and accepting the offer is

$$U_A(k, x_i) = (1 - t) U_D(k, x_i) + s.$$

Therefore, the maximum expected payoff from investing is

$$U(k, x_i) = \max\{U_D(k, x_i), U_A(k, x_i)\}.$$

The following lemma will be useful later on.

**Lemma 1** *Given that all other investors run on the bank if and only if their signal is below $k$, there exist two functions $k_1^*(k)$ and $k_2^*(k)$ such that an investor strictly prefers running if his private signal $x_i < k_1^*(k)$ and strictly prefers staying if $x_i > k_2^*(k)$. The functions $k_1^*(k)$ and $k_2^*(k)$ are given by*

$$k_1^*(k) = \inf\left\{k^* : U_D(k, k^*) \geq -\frac{s}{1-t}\right\},$$
$$k_2^*(k) = \sup\left\{k^* : U_D(k, k^*) \leq -\frac{s}{1-t}\right\}.$$

*Both $k_1^*(k)$ and $k_2^*(k)$ are weakly increasing in $k$.*

**Proof of Lemma 1.** The Lower Dominance Region assumption in Definition B.1 together with Limit Dominance (part (iii) of Assumption B.1) ensures that the functions $k_1^*(k)$ and $k_2^*(k)$ are well-defined. From the continuity of $U_D(k, x_i)$ in $x_i$ it follows that

$$U_D(k, k_1^*(k)) = U_D(k, k_2^*(k)) = -\frac{s}{1-t}.$$

On the one hand, for any $x_i < k_1^*(k)$ we have $U_D(k, x_i) < -\frac{s}{1-t}$ and $U_A(k, x_i) = (1-t)U_D(k, x_i) + s < 0$. Therefore, $U(k, x_i) = \max\{U_D(k, x_i), U_A(k, x_i)\} < 0$ and so the agent will not invest if he observes $x_i < k_1^*(k)$. On the other hand, for any $x > k_2^*(k)$ we have $U_D(k, x_i) > -\frac{s}{1-t}$ and $U_A(k, x_i) = (1-t)U_D(k, x_i) + s > 0$. Hence $U(k, x_i) = \max\{U_D(k, x_i), U_A(k, x_i)\} > 0$ and so the agent will invest after observing signal $x_i > k_2^*(k)$.

Because $U_D(k, x_i)$ is weakly decreasing in $k$, it is straightforward to show that both $k_1^*(k)$ and $k_2^*(k)$ are weakly increasing in $k$. ■

Now we can use Lemma 1 to prove the uniqueness of equilibrium using iterated deletion of dominated strategies. Denote the run strategy by $a_i(x_i)$. We want to show that a strategy survives $n$ rounds of iterated deletion of dominated strategies if and only if

$$a_i(x_i) = \begin{cases} 0 & \text{if } x_i < \underline{\xi}_n, \\ 1 & \text{if } x_i > \bar{\xi}_n, \end{cases}$$

where $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = \infty$. The terms $\underline{\xi}_n$ and $\bar{\xi}_n$ are defined inductively by $\underline{\xi}_{n+1} = k_1^*(\underline{\xi}_n)$ and $\bar{\xi}_{n+1} = k_2^*(\bar{\xi}_n)$.

Since $k^*(\xi)$ increases with $\xi$, it follows that $\underline{\xi}_n$ and $\bar{\xi}_n$ are (respectively) increasing and decreasing sequences. As $n \to \infty$, we have that $\underline{\xi}_n \to \underline{\xi}$ and $\bar{\xi}_n \to \bar{\xi}$; hence $\underline{\xi} = k_1^*(\underline{\xi})$ and

$\bar{\xi} = k_2^*(\bar{\xi})$. Therefore, both $\underline{\xi}$ and $\bar{\xi}$ must be solutions to

$$U_D(\xi, \xi) = -\frac{s}{1-t}.$$

Let $l = 1 - F\left(\frac{\xi-\theta}{\sigma}\right)$ be the total funding staying in the bank when the fundamentals are $\theta$ and investors follow threshold strategy $\xi$; then the preceding equality can be rewritten as

$$\int_0^1 \pi(\xi, l)\, \mathrm{d}l = -\frac{s}{1-t}. \tag{D.15}$$

By Strict Laplacian State Monotonicity (part (ii) of Assumption B.1), the left-hand side of (D.15) is continuous and strictly increasing in $\xi$. Moreover, at the two boundaries, $\int_0^1 \pi(\underline{\theta}, l)\, \mathrm{d}l \leq \pi(\underline{\theta}, 1) < -\frac{s}{1-t}$ and $\int_0^1 \pi(\bar{\theta}, l)\, \mathrm{d}l > 0 \geq -\frac{s}{1-t}$. Therefore, there exists a unique solution to (D.15): $\underline{\xi} = \bar{\xi} = \xi^*$. Note that, since $\xi^*$ is independent of $\sigma$, it is (by iterated deletion of dominated strategies) the unique run threshold in equilibrium.

Given the unique run threshold $\xi^*$, we can solve for the investors' participation strategy. An investor with $x_i \geq \xi^*$ participates in the intervention program if and only if $U_A(\xi^*, x_i) > U_D(\xi^*, x_i)$ or, equivalently, $U_D(\xi^*, x_i) < s/t$. Note that (a) $U_D(\xi^*, x_i)$ continuously increases with $x_i$, (b) $U_D(\xi^*, \xi^*) = -\frac{s}{1-t} < \frac{s}{t}$. If $s/t > \pi(\bar{x}, 1)$, then $U_D(\xi^*, x_i) < s/t$ for any $x_i > \xi^*$, and all investors with $x_i > \xi^*$ participate in the program. Therefore, the program is a full-participation program. If $s/t \leq \pi(\bar{x}, 1)$, there exists a unique solution $x_i = \eta^*(\sigma)$ such that

$$U_D(\xi^*, \eta^*(\sigma)) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\eta^*(\sigma) - \theta}{\sigma}\right) \pi\left(\eta^*(\sigma), 1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right) \mathrm{d}\theta = \frac{s}{t}. \tag{D.16}$$

Since $U_D(\xi^*, x_i)$ increases with $x_i$, for any signal $x_i > \eta^*(\sigma)$, an investor strictly prefers investing and declining the participation offer. The program is a partial-participation program.

Now suppose $s/t < \pi(\xi^*, 1)$. Denote the limiting participation threshold $\lim_{\sigma\to 0} \eta^*(\sigma) = \eta$. Since $U_D(\xi^*, x_i)$ increases with $x_i$ and since $U_D(\xi^*, \xi^*) < \frac{s}{t} = U_D(\xi^*, \eta^*(\sigma))$, we have $\eta^*(\sigma) > \xi^*$. It is then immediate that $\eta \geq \xi^*$. Next, we prove $\eta = \xi^*$ by way of contradiction. Suppose that $\eta > \xi^*$; then, taking $\sigma \to 0$ in the left-hand side of (D.16), we have

$$\lim_{\sigma\to 0} \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\eta^*(\sigma) - \theta}{\sigma}\right) \pi\left(\eta^*(\sigma), 1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right) \mathrm{d}\theta = \pi(\eta, 1) \geq \pi(\xi^*, 1) > \frac{s}{t},$$

which contradicts (D.16). Therefore, $\lim_{\sigma\to 0} \eta^*(\sigma) = \eta = \xi^*$. ■

**Proof of Proposition B.3.** For any target $\xi^* \in (\theta_0, \xi_0^*)$, the set of intervention programs with target $\xi^*$ is $\left\{(s,t) \mid t \in (0,1), s = -(1-t)\int_0^1 \pi(\xi^*, l)\,\mathrm{d}l\right\}$. Note that in this set $s/t = -\frac{1-t}{t}\int_0^1 \pi(\xi^*, l)\,\mathrm{d}l$ strictly decreases from $+\infty$ to 0 when $t$ increases from 0 to 1. From Proposition B.2 we know that whether the intervention program is a PPP or a FPP depends on the relationship between $s/t$ and $\pi(\bar{x}, 1)$. Let $t^*$ be the unique solution to $\pi(\bar{x}, 1) = -\frac{1-t}{t}\int_0^1 \pi(\xi^*, l)\,\mathrm{d}l$. For any intervention program with target $\xi^*$, the program is a FPP(PPP) if and only if $t < (\geq) t^*$. ■

**Proof of Proposition B.4.** Let $p_\theta$ and $p_{x|\theta}$ be the prior distribution of $\theta$ and the conditional distribution of $x$ on $\theta$ respectively. Denote the marginal distribution of signal $x$ by $p_x$. Therefore, by Bayes' rule, for a given investor with private signal $x$, the posterior distribution of $\theta$ can be written as $p_{\theta|x}(\theta) = p_\theta(\theta)p_{x|\theta}(x)/p_x(x)$. We expend the definition of $\eta^*(\sigma)$ to FPPs by making $\eta^*(\sigma) = \bar{x}$. For any program targeting $\xi^*$, the expected transfer from the policy maker to the investors can be written as

$$\begin{aligned}
\mathbb{E}_\theta \Gamma(\theta, s, t; \sigma) &= \int_{\underline{\theta}}^{\bar{\theta}} \int_{\xi^*}^{\eta^*(\sigma)} p_{x|\theta}(x) \left[\pi_A\left(x, 1 - F_{x|\theta}(\xi^*)\right) - \pi_D(x, 1 - F_{x|\theta}(\xi^*))\right] \mathrm{d}x\, p_\theta(\theta)\mathrm{d}\theta, \\
&= \int_{\xi^*}^{\eta^*(\sigma)} \int_{\underline{\theta}}^{\bar{\theta}} p_{\theta|x}(\theta) \left[\pi_A\left(x, 1 - F_{x|\theta}(\xi^*)\right) - \pi_D(x, 1 - F_{x|\theta}(\xi^*))\right] \mathrm{d}\theta\, p_x(x)\mathrm{d}x, \\
&= \int_{\xi^*}^{\eta^*(\sigma)} \left[U_A(\xi^*, x) - U_D(\xi^*, x)\right] p_x(x)\mathrm{d}x, \\
&= \int_{\xi^*}^{\bar{x}} \max\left\{U_A(\xi^*, x) - U_D(\xi^*, x), 0\right\} p_x(x)\mathrm{d}x.
\end{aligned}$$

Conditional on the same target threshold $\xi^*$, $U_D(\xi^*, x)$ doens't change with the intervention program. Now we only need show that for any $x > \xi^*$, $U_A(\xi^*, x)$ decreases in $t$. Taking derivative with respect to $t$, we have

$$\frac{\mathrm{d}}{\mathrm{d}t} U_A(\xi^*, x) = -U_D(\xi^*, x) + \frac{\mathrm{d}s}{\mathrm{d}t} = -U_D(\xi^*, x) + \int_0^1 \pi(\xi^*, l)\,\mathrm{d}l < 0.$$

The last inequality is because for any $x > \xi^*$, $U_D(\xi^*, x) > U_D(\xi^*, \xi^*) = \int_0^1 \pi(\xi^*, l)\,\mathrm{d}l$. Therefore, conditional on the same target threshold $\xi^*$, $\mathbb{E}_\theta \Gamma(\theta, s, t; \sigma)$ decreases in $t$. ■

**Proof of Proposition C.1.** The optimal strategy of investor $i$ in group $g$ conditional on his interim belief $p_i$ is

$$a_i^g = \begin{cases} 1_D & \text{if } \ p_i > p_2^*, \\ 1_A & \text{if } \ p_{1,g}^* < p_i \leq p_2^*, \\ 0 & \text{if } \ p_i \leq p_{1,g}^*; \end{cases}$$

here $p_{1,g}^* = \frac{1-s}{R_g - t - s}$ and $p_2^* = \frac{s}{t+s}$ are two threshold beliefs that satisfy $0 \leq p_{1,g}^* < p_2^* \leq 1$. Note that the threshold belief for early withdrawal $p_{1,g}^*$ is group-specific because different groups enjoy different returns from staying in the bank $R_g$ and hence have different run incentives. In contrast, the threshold belief for participating in the intervention program $p_2^*$ is the same for all groups as the intervention program is homogeneous to all investors.

Define a sequence $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ with $\underline{\xi}_n = \{\underline{\xi}_{n,g}\}_{g=1}^N$ and $\bar{\xi}_n = \{\bar{\xi}_{n,g}\}_{g=1}^N$ being the vectors of threshold signals such that a strategy survives $n$ rounds of iterated deletion of dominated strategies if and only if

$$a_i^g = 0 \quad \text{if } \ x_i < \underline{\xi}_{n,g}, \tag{D.17}$$

$$a_i^g \in \{1_D, 1_A\} \quad \text{if } \ x_i \geq \bar{\xi}_{n,g}. \tag{D.18}$$

Therefore, the sequence $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ can be expressed recursively as follows,

$$\underline{\xi}_{n+1,g} = \inf\{x : p_g(x; \underline{\xi}_n) \geq p_{1g}^*\}, \tag{D.19}$$

$$\bar{\xi}_{n+1,g} = \sup\{x : p_g(x; \bar{\xi}_n) \leq p_{1g}^*\}, \tag{D.20}$$

where $p_g(x, \xi)$ is a group-$g$ investor's belief about the probability of bank survival given that he receives a signal $x$ and other investors follow threshold strategy $\xi$.

For any $g \in \{1, ..., N\}$, starting with $\underline{\xi}_{0,g} = -\infty$ and $\xi_{0,g} = +\infty$, we can prove by induction that

$$-\infty = \underline{\xi}_{0,g} < \underline{\xi}_{1,g} \leq \cdots \leq \underline{\xi}_{n,g} \leq \cdots \leq \bar{\xi}_{n,g} \leq \cdots \leq \bar{\xi}_{1,g} < \bar{\xi}_{0,g} = +\infty. \tag{D.21}$$

Since any bounded monotonic sequence has a finite limit, take $n$ to $\infty$, we have the two sequences of vectors of threshold singals $\{\bar{\xi}_n\}_{n=0}^{\infty}$ and $\{\underline{\xi}_n\}_{n=0}^{\infty}$ converging to $\bar{\xi}$ and $\underline{\xi}$ correspondingly with

$$\bar{\xi} \geq \underline{\xi}. \tag{D.22}$$

Now we prove $\bar{\xi} = \underline{\xi}$ by contradiction. Suppose $\bar{\xi} > \underline{\xi}$. Let $h$ be the group such that $\bar{\xi}_h - \underline{\xi}_h = \max_g \left\{\bar{\xi}_g - \underline{\xi}_g\right\} > 0$. Recall that $\theta^*(\bar{\xi})$ denotes the fundamental threshold when investors follow run thresholds $\bar{\xi}$. By definition, as in (2), $\theta = \theta^*(\bar{\xi})$ is the solution to

$$\sum_{g=1}^{N} w_g m_g F_g \left( \frac{\bar{\xi}_g - \theta}{\sigma} \right) - \theta = 0. \tag{D.23}$$

Therefore, $\theta = \theta^*(\bar{\xi}) - (\bar{\xi}_h - \underline{\xi}_h)$ is the solution to

$$\sum_{g=1}^{N} w_g m_g F_g \left( \frac{\bar{\xi}_g - (\bar{\xi}_h - \underline{\xi}_h) - \theta}{\sigma} \right) - \theta - (\bar{\xi}_h - \underline{\xi}_h) = 0. \tag{D.24}$$

Similarly, $\theta^*(\underline{\xi})$ is the fundamental threshold when investors follow run thresholds $\underline{\xi}$, and $\theta = \theta^*(\underline{\xi})$ is the solution to

$$\sum_{g=1}^{N} w_g m_g F_g \left( \frac{\underline{\xi}_g - \theta}{\sigma} \right) - \theta = 0. \tag{D.25}$$

Let's compare (D.25) and (D.24). Since $\underline{\xi}_g \geq \bar{\xi}_g - (\bar{\xi}_h - \underline{\xi}_h)$ and $\bar{\xi}_h - \underline{\xi}_h > 0$, the left hand side of (D.25) is strictly larger than the left hand side of (D.24) for any given $\theta$. Given the left hand side of (D.25) is strictly decreasing in $\theta$, we must have $\theta^*(\underline{\xi}) > \theta^*(\bar{\xi}) - (\bar{\xi}_h - \underline{\xi}_h)$. Therefore,

$$\begin{aligned}
p_h(\bar{\xi}_h; \bar{\xi}) =& Pr[\theta > \theta^*(\bar{\xi}) | \bar{\xi}_h], \\
=& F_h \left( \frac{\bar{\xi}_h - \theta^*(\bar{\xi})}{\sigma} \right), \\
=& F_h \left( \frac{\underline{\xi}_h - (\theta^*(\bar{\xi}) - (\bar{\xi}_h - \underline{\xi}_h))}{\sigma} \right), \\
>& F_h \left( \frac{\underline{\xi}_h - \theta^*(\underline{\xi})}{\sigma} \right), \\
=& p_h(\underline{\xi}_h; \underline{\xi}).
\end{aligned}$$

However, (D.19) and (D.20) implies $p_h(\bar{\xi}_h; \bar{\xi}) = p_h(\underline{\xi}_h; \underline{\xi}) = p^*_{1,h}$. Contradiction. This implies $\bar{\xi} = \underline{\xi}$. That is, there exists a unique equilibrium in which investors follow run threshold $\bar{\xi} = \underline{\xi} = \xi^*(s,t)$.

Run thresholds $\xi^*(s,t) = \{\xi^*_g(s,t)\}_{g=1}^N$ and fundamental threshold $\theta^*(s,t)$ are jointly characterized by

$$\sum_{g=1}^{N} w_g m_g F_g \left( \frac{\xi_g^*(s,t) - \theta^*(s,t)}{\sigma} \right) = \theta^*(s,t), \tag{D.26}$$

$$F_g \left( \frac{\xi_g^*(s,t) - \theta^*(s,t)}{\sigma} \right) = p_{1,g}^* \ \ \forall g \in \{1, \ldots, N\}. \tag{D.27}$$

Plugging (D.27) into (D.26) we have

$$\theta^*(s,t) = \sum_{g=1}^{N} m_g w_g p_{1,g}^*, \tag{D.28}$$

$$\xi_g^*(s,t) = \sum_{g=1}^{N} m_g w_g p_{1,g}^* + \sigma F_g^{-1}\left(p_{1,g}^*\right) \quad \forall g \in \{1, \ldots, N\}. \tag{D.29}$$

The associated participation thresholds $\eta^*(s,t) = \{\eta_g^*(s,t)\}_{g=1}^N$ are characterized by

$$p_g(\eta_g^*(s,t); \xi^*(s,t)) = p_2^* \ \ \forall g \in \{1, \ldots, N\}. \tag{D.30}$$

Solving (D.30) yields

$$\eta_g^*(s,t) = \sum_{g=1}^{G} m_g w_g p_{1,g}^* + \sigma F_g^{-1}(p_2^*) \ \ \forall g \in \{1, \ldots, N\}. \tag{D.31}$$

Plugging in the expressions for the two threshold beliefs $p_{1,g}^* = \frac{1-s}{R_g - t - s}$ and $p_2^* = \frac{s}{t+s}$ completes the proof. ■

**Proof of Proposition C.2.** Proposition C.1 characterizes the equilibrium under a partial-participation program $(s,t)$. Under a PPP with $s = 1$ and $t \in (1, R_{min} - 1)$, it can be verified that, when $\sigma \to 0$, both $\xi_g^*$ and $\eta_g^*$ converge to 0 for any group $g$. Hence this PPP achieves the first-best outcome: for any fundamental $\theta > 0$, all investors take action $a_i = 1_D$ and exert effort; for any fundamental $\theta < 0$, all investors take action $a_i = 0$. The mass of participants in the program is zero (except when $\theta = 0$); hence for any continuous distribution of the fundamental, the program's ex ante cost of implementation converges to zero. ■