Intervention with Screening in Panic-Based Runs*

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Abstract

Policymakers frequently use guarantees to reduce panic-based runs in the financial system. We analyze a binary-action coordination game under the global games framework and propose a novel intervention program that screens investors based on their heterogeneous beliefs about the system's stability. This program attracts only investors who are at the margin of running, and their participation boosts all investors' confidence in the financial system. Compared with government guarantee programs, our proposed program is as effective at reducing panic runs yet features two advantages: it costs less to implement and is robust to moral hazard.

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1 Introduction

In the modern financial system, coordination failures among investors and financial institutions can lead to self-fulfilling panic "runs" that threaten the system's stability and generate huge welfare losses.[1] Policymakers worldwide have therefore devoted considerable effort and resources to intervention programs that aim to prevent panic among investors. For instance: during the Great Recession of 2007–2008, governments provided financial institutions with loans, explicit and implicit guarantees, and capital injections. These policies proved to be effective at restoring financial stability, but they were criticized on two grounds. First, implementing guarantee programs on such a large scale entailed large fiscal costs, which jeopardized the sustainability of sovereign debt and led – in many European countries – to a sovereign debt crisis (Acharya, Drechsler and Schnabl, 2014; Farhi and Tirole, 2018). Second, the policies were criticized for their vulnerability to moral hazard problems (Cooper and Ross, 2002; Allen et al., 2018).

Given the downsides of conventional interventions, a natural question is whether those drawbacks can be overcomed without reducing the effectiveness of measures intended to prevent self-fulfilling runs. We answer this question by analyzing a coordination game in the context of bank runs in a standard global games framework (Morris and Shin, 2003). In such environment, even when there is a minimal amount of information friction among bank investors, they hold diverse beliefs about the likelihood of bank failure. We therefore propose a type of subsidy–tax program, with voluntary participation, that screens agents based on their beliefs. Compared with conventional programs such as government guarantees, our program has two advantages. First, only a small group of pivotal investors – who matter the most for the coordination outcome – self-select into the program, which saves on implementation costs while achieving the same effectiveness. Second, the "monitoring" role of bank runs is preserved, which reduces the moral hazard problem of banks.

In the benchmark model, a continuum of investors simultaneously decide whether to run on a bank. A key aspect of this model is that the run decisions feature strategic complementarities. Since a bank fails if either its fundamentals are weak or a critical mass of investors run on the bank, an investor has more incentive to run if he expects others to do likewise. As in standard global games, each investor receives a noisy private signal about the bank's fundamentals and makes inferences about other investors' run decisions. In the unique equilibrium, investors follow a threshold strategy: they run if their private signals fall below a certain signal threshold. As a result, the bank defaults if its fundamentals fall

[1] Examples of coordination failures include – but are not limited to – bank runs (Diamond and Dybvig, 1983), financial market runs (Bernardo and Welch, 2004), debt rollover problems (Morris and Shin, 2004), and credit freezes (Bebchuk and Goldstein, 2011).

below a fundamental threshold. For a region of fundamentals just below that threshold, a bank fails because of coordination failures among investors, and it would have survived if no investors had run on the bank. So to improve social welfare, the policymaker seeks to lower the fundamental threshold and thereby minimize self-fulfilling bank runs.

Next, we allow the policymaker to offer all investors a subsidy–tax program with voluntary participation. An investor who accepts the offer and does not run on the bank will receive a subsidy if the bank fails and will pay a tax if the bank survives. We classify the responses to this intervention into three categories based on the tax-to-subsidy ratio. If a program has a high tax-to-subsidy ratio, then no investor participates and so the program has no effect on the run threshold; we refer to this category as *zero-participation programs* (ZPPs). If a program charges a nonpositive tax, then participating investors always receive nonnegative transfers from the policymaker; hence the program attracts all investors and we call it a *full-participation program* (FPP). Many existing intervention policies, including government guarantees, benefit all investors uniformly and thus fall into that category. In this paper we propose *partial-participation programs* (PPPs), which feature positive yet low tax-to-subsidy ratios such that some but not all investors participate. Conceptually, a PPP is similar to a costly insurance policy that charges a premium if the bank survives and provides protections if the bank fails. More importantly, PPPs screen investors based on their beliefs about the probability of bank failure.

Under a PPP, the most optimistic investors – who believe that bank failure is improbable – will stay in the bank and decline the participation offer because the insurance is considered to be too expensive. In contrast, the most pessimistic investors believe that the probability of bank failure is so high that staying in the bank is not an attractive option even if they are protected by the insurance. Thus only investors with intermediate beliefs will participate in the program. From their perspective, the insurance is provided at a reasonable price and staying in the bank is attractive given the protection offered by that insurance. We show that with PPP there is a unique equilibrium in which all investors follow a threshold strategy with two thresholds. An investor will invest without participating in the program if he receives a high signal, will invest and participate in the program if his signal is medium (i.e., between the two thresholds), and will not invest if he receives a low signal.

Although FPPs and PPPs can each reduce coordination failure, we derive the powerful result that any PPP incurs a lower expected cost of implementation than does any FPP that achieves the same coordination outcome.[2] This result is crucial for a policymaker with a limited budget and who must allocate her funding economically among several welfare-enhancing programs. Moreover, the PPP's cost savings relative to an FPP comes from both the extensive and the intensive margins. On the extensive margin, a PPP excludes the most optimistic investors; hence the policymaker subsidizes fewer investors than under a FPP. On the intensive margin, a PPP collects taxes from participants when the bank survives, which partially offsets the cost of providing subsidies in expectation.

To understand intuitively how PPPs can reduce coordination failure at a minimal cost, it is useful to go through the following thought process. Start with the original threshold equilibrium without intervention programs. In this equilibrium, investors with signals slightly below the run threshold would run on the bank. A PPP provides these investors with protection against bank failure and incentivizes them to stay in the bank. Anticipating that the introduction of a PPP will incentivize more investors to stay in the bank, all investors have greater confidence in the bank's survival. This persuades investors with even lower signals to accept the PPP offer and stay, which is again anticipated by the investors and further strengthens their incentive to stay in the bank. Repeating this thought process, the PPP-induced incentive to stay is amplified by higher-order beliefs and so coordination failures can be substantially reduced in equilibrium. At the same time, given that all investors are more optimistic about the bank's survival, the downside protection offered by the PPP becomes less appealing; hence, in equilibrium, the mass of investors who accept the offer is small.

In addition to the advantage of lower implementation costs, PPPs are also more robust (than are FPPs) to moral hazard problems. We establish this robustness by modifying the model to incorporate moral hazard (cf. Diamond and Rajan, 2001) and then comparing government guarantee programs – the least costly FPP with zero tax and positive subsidy – with PPPs. More specifically, the banker is allowed to choose an effort level after the policymaker announces the intervention program but before investors make their run decisions. The banker enjoys a private benefit from shirking, but shirking reduces investors' payoffs from staying in the bank and so increases the likelihood of bank runs. Because the banker receives a reward if the bank survives, bank runs play a monitoring role by increasing the cost of shirking. So without intervention programs, the monitoring effect of bank runs encourages the banker to exert a welfare-maximizing effort.

[2] The result of Diamond and Dybvig (1983) that government guarantees are costless to implement relies on the assumptions that (i) there are no information frictions and (ii) runs are driven only by panic and not by fundamentals. As long as information frictions exist, government guarantees – or any other FPPs – are costly to implement because some investors would mistakenly stay in a failing bank and ask for subsidies from the policymaker.

Government guarantee programs reduce an investor's "skin in the game" and makes his run decision insensitive to the banker's effort choice. Hence government guarantee programs hinder the monitoring role of bank runs and fail to limit shirking by the banker. In contrast, a PPP can achieve the same coordination outcome while preserving the monitoring role of bank runs. The reason is that investors internalize the cost of participating in a PPP. If the banker shirks, which reduces investors' payoffs from staying in the bank, then the tax charged by the PPP becomes so costly that no investor participates in the program. Thus the PPP acts as a switching mechanism: it deters bank runs if the banker exerts effort; but it turns into a ZPP (and so becomes inoperative) if the banker shirks. Therefore, PPPs can reduce coordination failure without inducing moral hazard.

Our proposed partial-participation programs can be applied to a wide range of applications beyond bank runs; examples include credit market freezes (Bebchuk and Goldstein, 2011) and asset market freezes (Bernardo and Welch, 2004). In some of these applications, moral hazard arises not at the aggregate level but rather at the individual level. In the context of a credit freeze, for instance, banks abstain from lending in anticipation of low aggregate credit supply and low returns from lending. Government guarantees encourage bank lending. Yet they reduce banks' skin in the game and thus also their incentive to screen and monitor borrowers. Unlike the moral hazard in our benchmark model, banks' shirking in credit assessment and monitoring only reduces their own profits from lending and does not directly spill over to other banks. In other words, the moral hazard problem occurs at individual level. To demonstrate the robustness of PPPs to individual moral hazard, we modify the benchmark model and assume that an investor can earn a private benefit from shirking, which reduces the success probability of his own investment. Since FPPs (e.g., government guarantees) and PPPs both subsidize investors' losses, they lead to shirking by participating investors. However, fewer investors participate in PPPs than in FPPs. It is important to bear in mind that, for the most optimistic investors, the tax component of a PPP is so costly that they optimally decline the offer – which incentivizes them to exert effort. So under PPPs, the individual moral hazard problem is limited to medium-belief investors, the mass of whom approaches zero in the limit of vanishing information frictions. In the limit, then, there are PPPs that can restore the first-best outcome; yet no government guarantee program can do so owing to the welfare loss resulting from individual moral hazard.

Literature To the best of our knowledge, our proposed partial-participation programs that screen investors based on their beliefs is novel to the literature. In particular, we demonstrate two main advantages of such programs: cost efficiency and robustness to moral hazard. Similar to our mechanism, several papers explore policies that target a specific group of agents to reduce coordination failure. For example, Sakovics and Steiner (2012) and Choi (2014) analyze coordination games with ex ante heterogeneous agents and argue that the optimal subsidy schedule is to target a certain type of investors.[3] Our proposed PPPs contrast these targeted intervention programs in mainly three aspects. First, we show that it is cost efficient to screen agents based on their interim beliefs rather than ex ante characteristics. In other words, policymakers should target interim rather than ex ante "pivotal" investors. Second, implementing a targeted intervention program requires the policymaker to correctly identify each investor's type, which can entail additional information acquisition costs. In contrast, our proposed PPPs incentivize "pivotal" investors to self-reveal their types, and the policymaker can simply offer a uniform intervention policy to all investors. Lastly, these papers focus on direct subsidies without a tax charge, which is a special type of the FPPs in our model. Hence the policy space that we consider is more general.

Our paper also contributes to the vast literature on panic-based bank runs and policies to enhance financial stability dating back to Diamond and Dybvig (1983). Although government-guarantee type of programs such as deposit insurance are proposed as effective intervention to reduce coordination failures among bank investors, the literature has also pointed out their limitations. Firstly, large-scale government guarantees link the stability of banks and sovereigns and allow bank runs to jeopardize the fiscal health of a country (Acharya, Drechsler and Schnabl, 2014; Leonello, 2018; Farhi and Tirole, 2018). Moreover, such programs are vulnerable to moral hazard problems (Cooper and Ross, 2002; ?; Allen et al., 2018).[4] Compared to government guarantees, our proposed program overcomes these two drawbacks without compromising the effectiveness of preventing panic runs.

In terms of methodology, our model builds on the global games literature pioneered by Carlsson and Van Damme (1993). Morris and Shin (2003) review the commonly applied set-up and applications of global games. Our benchmark model in Section 2 is a special case with binary payoffs, and we generalize the payoff structure in Appendix B. Researchers have applied global games techniques to analyze coordination failures in a variety of contexts; these include, among others, bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005), currency attacks (Morris and Shin, 1998), credit freezes (Bebchuk and Goldstein, 2011), debt rollovers (Morris and Shin, 2004; He and Xiong, 2012), and financial market

[3]Similar in spirit, within the contracting literature, Segal (2003) and Bernstein and Winter (2012) analyze situations in which a principal offers contracts to a group of agents, whose trade with the principal generates externalities on one another. They show that the optimal strategy for the principal is to *divide and conquer*: subsidize some of the agents so that they take the principal's desired action; this incentivizes other agents to take the same action and allows the principal to extract more surplus from agents who are not subsidized.

[4]See Demirgüç-Kunt and Detragiache (2002); Demirgüç-Kunt and Huizinga (2004); Ioannidou and Penas (2010) for empirical evidence that deposit insurance reduces monitoring by bank investors and increases bank risk-taking.

runs (Bernardo and Welch, 2004; Brunnermeier and Pedersen, 2009). Although we focus on bank runs, our model has a wide range of applications. As long as the policymaker can observe agents' actions and can condition the intervention program on those actions, implementing our proposed program can reduce coordination failures.

From a theoretical perspective, the study that is the most closely related to ours is Morris and Shadmehr (2017), who analyze the reward schemes that a revolutionary leader can offer in order to elicit effort from citizens. The ideal reward scheme also screens citizens based on their beliefs. Yet they consider mandatory reward schemes, whereas we focus on intervention programs with voluntary participation. A more crucial distinction is that, while Morris and Shadmehr (2017) assume that any reward scheme can be implemented at zero cost, we explicitly model and target to minimize the cost of intervention. Two other related works are those of Cong, Grenadier and Hu (2020) and Basak and Zhou (2020), who explore intervention policies under dynamic settings. In both of these papers, the policymaker targets a subset of agents in each period; successful coordination in previous periods serves as a public signal that upholds agents' confidence in subsequent periods. Our paper considers a static coordination game, in which investors' confidence is boosted by their inferences about other investors' reactions to an intervention program. Therefore, the effectiveness of our proposed intervention programs does not rely on a dynamic information structure.

The rest of our paper proceeds as follows. In Section 2, we present a benchmark model of a binary-action investment game and introduce intervention policies that can reduce coordination failures. Sections 3 and 4 compare the proposed program with government guarantee–type programs in terms of implementation cost and robustness to moral hazard. Two extensions of the benchmark model are discussed in Section 5. We conclude in Section 6 with a brief summary of our study's results and limitations.

2 Benchmark Model

In this section, we analyze a benchmark model in the context of classic panic-based bank runs. We first describe the environment without intervention programs and show that coordination failure among investors can lead to welfare losses. Then we introduce intervention programs and demonstrate how they can help reduce the likelihood of such failure.

2.1 Bank Runs

There are three periods, $t \in \{0, 1, 2\}$. At $t = 0$, a financial institution (for short, a bank) collects one unit of capital from a unit mass of infinitesimal investors, indexed by $i \in [0, 1]$, in the form of demandable debt. The bank then makes long-term investments that mature at $t = 2$ and can be liquidated prematurely, at a cost, at $t = 1$.[5]

At $t = 1$, each investor i makes a withdrawal decision $a_i \in \{0, 1\}$: $a_i = 0$ if investor i withdraws his funds early, or $a_i = 1$ if he stays in the bank until $t = 2$. To accommodate early withdrawals, the bank must undertake a costly liquidation, at $t = 1$, of its long-term investments. A bank fails if it has insufficient assets to fulfill early withdrawals. Investors who withdraw their funds early ($a_i = 0$) are guaranteed to receive back their initial investments. Investors who stay ($a_i = 1$) receive nothing if the bank fails but enjoy returns of $R > 1$ if the bank survives. In what follows, we normalize investor i's payoff from early withdrawal to 0. Hence his payoff from staying is

$$\pi(\theta, l) = \begin{cases} R - 1 & \text{if } 1 - l \leq \theta, \\ -1 & \text{if } 1 - l > \theta. \end{cases} \tag{1}$$

Here $1 - l = \int_0^1 \mathbb{1}_{\{a_i = 0\}} \, \mathrm{d}i$ is the aggregate early withdrawal at $t = 1$, and θ represents the fundamentals that determine the maximum amount of early withdrawals that the bank can fulfill. Economically, θ reflects the quality of the bank's assets as well as the market conditions under which the bank liquidates its long-term investments. When the bank experiences total early withdrawals $1 - l$ in excess of its capacity (as determined by the fundamentals θ), the bank fails.

In the benchmark model we focus, for the sake of clarity, on a simple binary payoff structure.[6] In Appendix B, we consider a generalized continuous payoff structure and demonstrate the robustness of our main results to that alternative. It is important that the payoff structure of investors features strategic complementarities (as in Diamond and Dybvig, 1983); hence an investor's incentive to withdraw his funds early increases with the mass of other investors who do so.

In terms of information structure, we follow the standard global games literature and make the following assumptions. At $t = 0$, the fundamentals term θ is drawn from a uniform distribution with support $[\underline{\theta}, \bar{\theta}]$ and is not observable to investors.[7] At $t = 1$, each investor i receives a private and noisy signal, $x_i = \theta + \sigma\varepsilon_i$, about the fundamentals; here σ represents the magnitude of information friction and ε_i is independent and identically distributed (i.i.d.)

[5] Besides financial institutions, the environment described here encompasses any firm that relies on short-term debt.

[6] The payoff structure we adopt resembles the set-up of Rochet and Vives (2004); in that study, fund managers – just like the bank investors in our model – make run decisions and earn binary payoffs.

[7] We assume a uniform prior in order to obtain an analytical solution to the coordination game. It can be viewed as an approximation of a continuous prior distribution function when the magnitude of information friction is small.

with a continuous and strictly increasing cumulative distribution function $F(\varepsilon)$, the support of which is $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Furthermore, we assume that $\underline{\theta} < -\sigma$ and $\bar{\theta} > 1 + \sigma$. Under these assumptions, there exist two dominance regions of signals, $\left[-\underline{\theta} - \frac{1}{2}\sigma, \underline{x}\right)$ and $\left(\bar{x}, \bar{\theta} + \frac{1}{2}\sigma\right]$. Here $\underline{x}$ and $\bar{x}$ are such that

$$\Pr[\theta \geq 1 \mid x = \bar{x}] = \frac{1}{R},$$
$$\Pr[\theta \geq 0 \mid x = \underline{x}] = \frac{1}{R}.$$

We can see intuitively that an investor who receives a *high* private signal $\bar{x}$ is indifferent between running or not, even when all other investors run on the bank ($l = 0$). So if an investor receives a signal $x > \bar{x}$ then his dominant strategy is to stay in the bank. An investor who receives a *low* signal $\underline{x}$ is, analogously, indifferent between running or not, even when all other investors stay in the bank ($l = 1$). In this case, if $x < \underline{x}$ then withdrawing early is the dominant strategy.

2.2 Equilibrium without Intervention

We now analyze the equilibrium without intervention and identify the inefficiencies due to coordination failure. Proposition 1 characterizes the equilibrium.

Proposition 1 *Without intervention, there is a unique equilibrium in which all agents follow the same strategy:*

$$a_i(x_i) = \begin{cases} 1 & \text{if } x_i \geq \xi_0^*, \\ 0 & \text{if } x_i < \xi_0^*; \end{cases}$$

here $\xi_0^* = \frac{1}{R} + \sigma F^{-1}\left(\frac{1}{R}\right)$.

Given any realization of θ, we can use this proposition to calculate the total early withdrawal $1 - l$ and thus to predict the coordination outcomes. Because all investors follow the same threshold strategy in equilibrium, the coordination outcome also features a threshold above which the bank survives. Let $\theta^*(\xi)$ denote the fundamental threshold when all agents follow the threshold strategy ξ; then $\theta^*(\xi)$ is defined by

$$F\left(\frac{\xi - \theta^*(\xi)}{\sigma}\right) = \theta^*(\xi). \tag{2}$$

That is to say: at the fundamental threshold, the amount of early withdrawals reaches the bank's maximum capacity. So in equilibrium, the fundamental threshold is given by

$$\theta^*(\xi_0^*) = \frac{1}{R}.$$

We can therefore divide the fundamental realizations into three regions, as shown in Figure 1.

Figure 1: Coordination Outcome

In the middle region $\theta \in [0, 1/R)$, if no investor withdraws his funds early then the bank survives. Yet in equilibrium, enough investors form self-fulfilling beliefs that others will run on the bank, and the bank indeed suffers severe runs and fails. In this region, a unit mass of early withdrawals leads to a social welfare loss of $R - 1 > 0$. Hence the first-best scenario is for all investors to stay in the bank as long as $\theta \geq 0$. We shall next introduce intervention programs and show how they lower the fundamental threshold, which reduces the inefficiencies caused by coordination failure among investors.

2.3 Intervention Programs

Having characterized the equilibrium in the game without intervention, we are prepared to describe the subsidy–tax intervention program that a policymaker can use to minimize panic-based bank runs.

At $t = 0$, the policymaker announces an intervention program (s, t) before the realization of fundamentals θ. The program is offered only to investors who choose to stay in the bank, and their participation in the program is voluntary.[8] At $t = 1$, investors observe their respective private signals x_i and decide whether to withdraw their funds early – and, if not, whether to accept or decline the offer (s, t). Investor i's action space is then enlarged to $a_i \in \{0, 1_A, 1_D\}$: $a_i = 0$ if investor i withdraws his funds early; $a_i = 1_A$ if he stays in the bank and accepts the offer; and $a_i = 1_D$ if he stays in the bank and declines the offer. When $a_i = 1_D$ or $a_i = 0$, investor i's payoff remains as specified in Section 2.1. In particular, the payoff from early withdrawal ($a_i = 0$) is simply zero and the payoff from staying and

[8] An implicit assumption here is that the policymaker can observe and contract on investors' actions. As shown in Bond and Pande (2007), if the policymaker cannot observe individual actions then her ability to use subsidy–tax schemes as a coordination device is extremely limited. One implication is that the intervention program discussed here cannot be applied to currency attacks (Morris and Shin, 1998), in which the actions of agents are difficult to trace. Despite this limitation, there remains a wide range of real-world applications. In Section 5, we discuss two representative examples.

declining the offer ($a_i = 1_D$) is

$$\pi_D(\theta, l) = \pi(\theta, l).$$

When $a_i = 1_A$, investor i receives a subsidy s if the bank fails or must pay a tax t if the bank survives. The expected payoff from staying and accepting the offer ($a_i = 1_A$) can therefore be written as

$$\pi_A(\theta, l) = \begin{cases} R - 1 - t & \text{if } 1 - l \leq \theta, \\ -1 + s & \text{if } 1 - l > \theta. \end{cases}$$

In essence, the offer (s, t) amounts to a costly insurance; t is the insurance premium and $s + t$ is the coverage when the bank fails. The subsidy s reduces agents' exposure to the risk of a coordination failure and encourages them to stay in the bank. The tax t serves two purposes. First, as we show in Section 3, it reduces the cost of implementing the intervention program by directly collecting taxes when the bank survives and indirectly discouraging optimistic agents from participating in the program. Second, as established in Section 4, the tax deters moral hazard problems by keeping each participating investor's skin in the game. Because investors receive binary payoffs, it is sufficient to focus on intervention programs (s, t) involving lump-sum transfers between the policymaker and participating investors.[9] In what follows, we restrict $s \geq 0$ and $t \leq R$ because of investors' limited liability. In addition, we also impose $s \leq 1$ since intervention programs with $s > 1$ incentivize investors to say in the bank even when it fails for sure ($\theta < 0$).

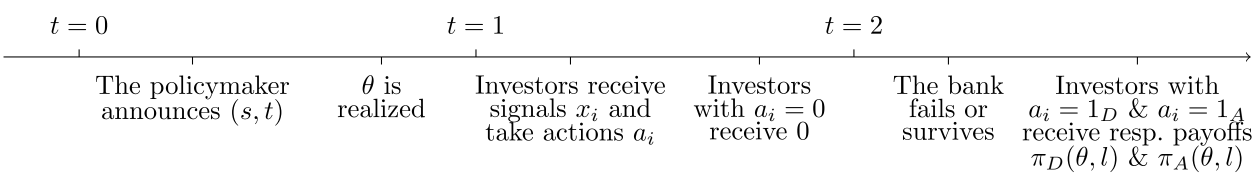


Figure 2: Timeline

Figure 2 summarizes the sequence of events in the game with an intervention program. We can immediately see one advantage of the intervention program (s, t): it does not depend on the fundamentals θ. Angeletos, Hellwig and Pavan (2006) point out the informational effect of state-contingent policies – namely, they can reveal the policymaker's private information about the state of the economy. This informational effect not only undermines the efficiency of intervention policies but also leads to multiple equilibria in global games. Yet the intervention programs analyzed in this paper are free of such problems, since the an-

[9] In Appendix B we generalize the payoff structure to be continuous. In a similar spirit, the intervention program then consists of a lump-sum subsidy and a tax that is proportional to investors' payoffs.

nouncement of a program does *not* convey any information about θ that the policymaker may possess. Moreover, a policymaker can implement the program without having any superior information about the fundamentals θ.

Even though intervention program (s,t) is specified in a subsidy–tax form, it can be interpreted as other forms of intervention with transfers – contingent on the coordination result – between policymaker and investors. For example, a government guarantee–type program that promises to cover bank failure losses up to s is equivalent to a subsidy–tax program $(s,0)$.

2.4 Equilibrium with Intervention

We now analyze the equilibrium with intervention and demonstrate how intervention programs reduce coordination failure. Under intervention program (s,t), each investor i has three choices: $a_i \in \{0, 1_A, 1_D\}$. Note that investors' participation decisions – whether $a_i = 1_D$ or 1_A – have no impact on the aggregate early withdrawal $1 - l = \int_0^1 \mathbb{1}_{\{a_i=0\}}\,\mathrm{d}i$ or on the bank's survival. As a result, when analyzing an investor's equilibrium strategies it is sufficient to condition on *other* investors' withdrawal strategies. We facilitate discussion by defining p_i, the interim belief of investor i, as his estimate of the probability that the bank will survive; that estimate is conditional on (a) his private signal x_i and (b) other agents' strategies $a_{-i}(x)$:

$$p_i = \Pr[1 - l \leq \theta \mid x_i; a_{-i}(x)].$$

Given his belief, investor i's expected payoffs from $a_i = 1_D$ and $a_i = 1_A$ are (respectively)

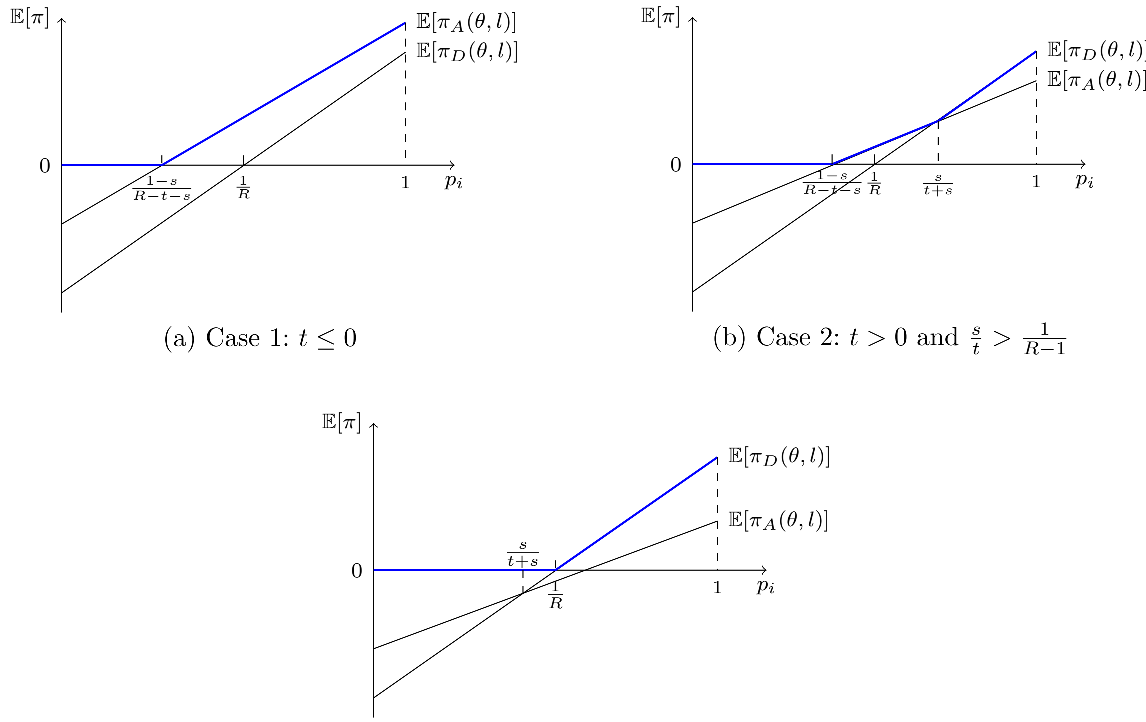
$$\mathbb{E}[\pi_D(\theta,l) \mid x_i; a_{-i}(x)] = p_i R - 1 \quad \text{and} \tag{3}$$

$$\mathbb{E}[\pi_A(\theta,l) \mid x_i; a_{-i}(x)] = p_i(R - t - s) - (1 - s); \tag{4}$$

his expected payoff from early withdrawal ($a_i = 0$) is zero.

Figure 3 plots expected payoffs as a function of the interim belief p_i for three action choices; in each graph, the blue line corresponds to the maximum payoff. The three cases reflect distinct levels of the intervention program's generosity.

In the first case, shown in panel (a), if $t \leq 0$ then the intervention program is a "free lunch". That is, an investor who accepts the offer will receive a nonnegative transfer from the government regardless of the coordination outcome. It follows that accepting dominates declining, so all investors who stay in the bank will accept the offer; thus we have a so-called *full-participation program* (FPP). Absent intervention, the belief threshold for early withdrawal is $\frac{1}{R}$, at which point an investor is indifferent between withdrawing early or not.



(a) Case 1: $t \leq 0$

(b) Case 2: $t > 0$ and $\frac{s}{t} > \frac{1}{R-1}$

(c) Case 3: $t > 0$ and $\frac{s}{t} \leq \frac{1}{R-1}$

Figure 3: Expected Payoffs and Interim Beliefs

With a full-participation program, the threshold falls to $\frac{1-s}{R-t-s}$. In the third case, illustrated in panel (c) of the figure, if $t > 0$ and $\frac{s}{t} \leq \frac{1}{R-1}$ then declining the offer dominates accepting it. We call this a *zero-participation program* (ZPP). Here the threshold belief with intervention is simply the same as without intervention: $\frac{1}{R}$.

The most interesting case is shown in Figure 3(b). Here $t > 0$ and $\frac{s}{t} > \frac{1}{R-1}$, so an investor who participates in the program is essentially purchasing insurance against bank failure. If the bank survives, a participating investor is taxed an amount t; if the bank fails, the investor receives compensation s. For investors with extreme beliefs, the insurance is not valuable. On the one hand, investors with optimistic beliefs $p_i > \frac{s}{t+s}$ estimate a high probability of bank survival and so they would stay in the bank even without the intervention program. Furthermore, given the high probability of paying t and low probability of receiving s, this "insurance" (intervention program) is overpriced. Therefore, the optimal action of such an investor is to stay in the bank and decline the offer. On the other hand, investors with pessimistic beliefs $p_i < \frac{1-s}{R-t-s}$ estimate a high probability of bank failure. As long as the intervention program does not guarantee zero loss upon bank failure ($s < 1$), the "insurance" does not provide enough coverage for them to stay in the bank. Hence their optimal action for such investors is to run on the bank ($a_i = 0$).

In contrast, the insurance generates value for investors who have medium beliefs $p_i \in \left[\frac{1-s}{R-t-s}, \frac{s}{t+s}\right]$ because they are uncertain about the coordination outcome. Since not all staying investors participate in the program, we call this type of program a *partial-participation program* (PPP). It is worth noting that, in effect, partial-participation programs convince investors to stay in the bank when their beliefs $p_i \in \left[\frac{1-s}{R-t-s}, \frac{1}{R}\right]$ – that is, investors who would run on the bank *without* an intervention program. As a consequence, the threshold belief is lowered to $\frac{1-s}{R-t-s}$. Both full- and partial-participation programs lower the threshold belief and discourage bank runs; yet as we show in Section 3, these two program types differ substantially in implementation costs.

Our next proposition characterizes the equilibrium in the presence of a subsidy–tax intervention program (s, t).

Proposition 2 *When the policymaker offers a subsidy–tax intervention program (s, t) with $s \in [0, 1]$, the game has a unique equilibrium.*

(i) *(Full-participation programs) If $t \leq 0$, then the equilibrium stategy of investor i is*

$$a_i = \begin{cases} 1_A & \text{if } x_i \geq \xi^*(s,t), \\ 0 & \text{if } x_i < \xi^*(s,t), \end{cases}$$

where

$$\xi^*(s,t) = \frac{1-s}{R-t-s} + \sigma F^{-1}\left(\frac{1-s}{R-t-s}\right).$$

(ii) *(Partial-participation programs) If $t > 0$ and $\frac{s}{t} > \frac{1}{R-1}$, then the equilibrium stategy of investor i is*

$$a_i = \begin{cases} 1_D & \text{if } x_i \geq \eta^*(s,t), \\ 1_A & \text{if } \xi^*(s,t) \leq x_i < \eta^*(s,t), \\ 0 & \text{if } x_i < \xi^*(s,t); \end{cases}$$

here $\eta^*(s,t)$ is the participation threshold and

$$\xi^*(s,t) = \frac{1-s}{R-t-s} + \sigma F^{-1}\left(\frac{1-s}{R-t-s}\right),$$
$$\eta^*(s,t) = \frac{1-s}{R-t-s} + \sigma F^{-1}\left(\frac{s}{t+s}\right).$$

(iii) *(Zero-participation programs) If $t > 0$ and $\frac{s}{t} \leq \frac{1}{R-1}$, then the equilibrium stategy of* *investor i is*

$$a_i = \begin{cases} 1_D & \text{if } x_i \geq \xi^*(s,t), \\ 0 & \text{if } x_i < \xi^*(s,t), \end{cases}$$

where

$$\xi^*(s,t) = \xi_0^* = \frac{1}{R} + \sigma F^{-1}\left(\frac{1}{R}\right).$$

The proof of Proposition 2 is given in Appendix C. For FPPs (case (i)) and ZPPs (case (iii)), staying investors either all accept or all decline the offer. Therefore, the equilibrium analyses directly follow Proposition 1. It is interesting that investors' responses to PPPs (case (ii)) are nontrivial. Recall that such programs incentivize investors with beliefs $p_i \in \left[\frac{1-s}{R-t-s}, \frac{1}{R}\right]$ to stay in the bank, reducing the aggregate early withdrawal $1-l$. In turn, this reduction in $1-l$ strengthens the incentive of all investors to stay in the bank; that outcome further incentivizes investors to stay and thus lowers $1-l$. This virtuous cycle amplifies the effectiveness of PPPs at reducing coordination failure. Next we outline the equilibrium analyses and discuss the intuition for this amplification effect.

It turns out that we can focus on threshold strategies.[10] Investor i cares only about whether other investors run on the bank; he is not concerned about whether they participate in the intervention program. We therefore use the language that another investor j follows a *threshold run strategy* with threshold k provided he runs on the bank ($a_j = 0$) if and only if $x_j < k$. Then we can rewrite the interim belief $p_i = p(x_i; k)$ as a function of investor i's private signal x_i and the run threshold for all other investors k:

$$p(x_i; k) = \Pr[\theta > \theta^*(k) \mid x_i] = F\left(\frac{x_i - \theta^*(k)}{\sigma}\right), \tag{5}$$

where $\theta^*(k)$ is the fundamental threshold for bank survival and satisfies $F\left(\frac{k-\theta^*(k)}{\sigma}\right) = \theta^*(k)$ as defined in (2). Observe that $p(x_i; k)$ increases with x_i but decreases with k. It makes sense that (a) a high private signal x_i indicates a high realization of fundamentals θ and (b) a low run threshold k is suggestive of a low aggregate early withdrawal $1-l$; both imply a high likelihood of bank survival.

Figure 4 illustrates how the effectiveness of PPPs is amplified by investors' higher-order beliefs. In each iteration, the lower and upper axes represent (respectively) the signal received by an investor and his corresponding belief. Start from the threshold run strategy ξ_0^*, which is the equilibrium threshold without intervention. An investor who believes that other investors all adopt run threshold ξ_0^* is willing to lower his own run threshold to ξ_1^* by participating

[10] See the proof of Proposition 2 for details.

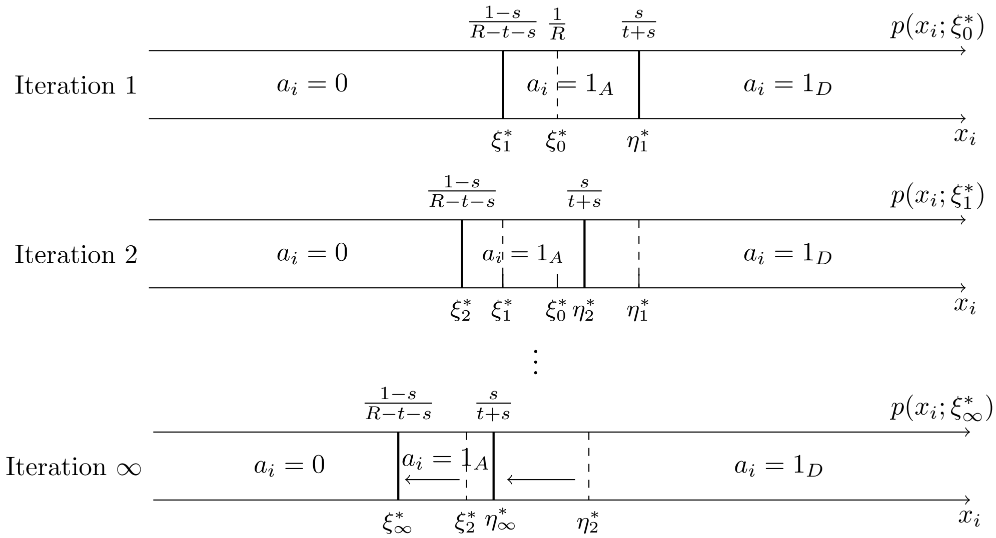


Figure 4: Amplification via Higher-Order Beliefs

in the intervention program. So in iteration 2, investors update their beliefs knowing that other investors have adopted a lower run threshold $\xi_1^* < \xi_0^*$. Since $p(x_i; k)$ decreases with k, it follows that all investors – given their private signals fixed – become more optimistic in iteration 2 about bank survival. Intuitively, all agents realize the intervention program will incentivize more investors to stay, which reduces early withdrawals l and stabilizes the bank; hence they are willing to lower their run threshold further, to ξ_2^*. In iteration 3, all investors become similarly more optimistic because they are aware that investors with signals between ξ_1^* and ξ_2^* are incentived to stay in the bank; therefore, investors lower their investment threshold further to ξ_3, and so forth. Yet as investors become more optimistic about the bank's stability, the intervention program becomes less appealing, which implies a decreasing sequence of participation thresholds $\{\eta_n^*\}_{n=1}^{\infty}$. With an infinite number of iterations, both the investment threshold and the participation threshold decrease markedly in equilibrium. Hence the mass of investors accepting the offer becomes fairly small while the probability of bank runs is appreciably reduced. We call these program participants *pivotal* investors because their payoffs determine the equilibrium fundamental threshold θ^*.

Both full- and partial-participation programs achieve a fundamental threshold above which the bank survives,

$$\theta^*(\xi^*(s,t)) = \frac{1-s}{R-t-s}, \tag{6}$$

which is lower than the threshold $\theta^*(\xi_0^*)$ *without* intervention. Therefore, offering this intervention program successfully reduces the region of inefficient bank runs. If the policymaker sets $s = 1$ and $t < R - 1$ then the fundamental threshold can be reduced to zero, which entirely eliminates the region of inefficient bank runs; see Figure 5.

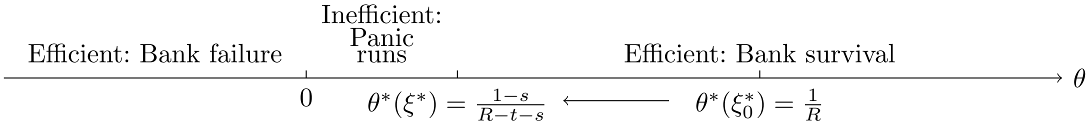


Figure 5: Coordination Outcome after Intervention

3 Cost of Intervention

In this section, we compare the welfare achieved by different intervention programs and show that partial-participation programs dominate full-participation programs. In Section 3.1, we define the policymaker's welfare maximization problem. The key is to find a cost-minimizing intervention program that achieves a target fundamental threshold. In Section 3.2, we prove that achieving any such target always incurs a lower implementation cost under a PPP than under an FPP. Moreover, partial-participation programs save costs on both the extensive and the intensive margin.

3.1 Welfare Maximization

With intervention program (s, t), social welfare is equal to investors' total investment payoffs before tax and subsidy *minus* the cost of implementing the intervention program.

Following realization of the fundamentals θ, investors' total payoffs before tax and subsidy can be written as

$$W(\theta, \hat{\theta}; \sigma) = \begin{cases} -\left[1 - F\left(\frac{\hat{\theta}+\sigma F^{-1}(\hat{\theta})-\theta}{\sigma}\right)\right] & \text{if } \theta < \hat{\theta}, \\ (R-1)\left[1 - F\left(\frac{\hat{\theta}+\sigma F^{-1}(\hat{\theta})-\theta}{\sigma}\right)\right] & \text{if } \theta \geq \hat{\theta}, \end{cases} \tag{7}$$

where $\hat{\theta} = \theta^*(\xi^*(s,t))$ is the equilibrium fundamental threshold with intervention program (s, t). In other words, the equilibrium fundamental threshold is a sufficient statistic for investors' payoffs; so conditional on the fundamental threshold, the tax t and subsidy s of an intervention program do not matter. Hence the policymaker can maximize social welfare by identifying the least costly program (s, t) that implements a socially optimal fundamental threshold $\hat{\theta}$.

Formally, the policymaker chooses an intervention program (s, t) to solve the following welfare maximization problem:

$$\max_{s,t} \mathbb{E}_\theta[W(\theta,\hat{\theta};\sigma)] - \mathbb{E}_\theta[C(\theta,s,t;\sigma)],$$

$$\text{s.t.} \quad \hat{\theta} = \theta^*(\xi^*(s,t)), \tag{8}$$

$$0 \leq s \leq 1, \; t \leq R. \tag{9}$$

Here $C(\theta, s, t; \sigma)$ is the ex post cost of providing intervention program (s, t) after the realization of fundamentals θ. We will specify $C(\theta, s, t; \sigma)$ in Section 3.2. The problem can be solved in two steps. First, for any target fundamental threshold $\hat{\theta}$, the policymaker selects the least costly program (s, t) that implements $\hat{\theta}$:

$$C_{min}(\hat{\theta};\sigma) = \inf_{s,t} \mathbb{E}_\theta[C(\theta,s,t;\sigma)],$$

$$\text{s.t.} \quad \hat{\theta} = \theta^*(\xi^*(s,t)),$$

$$0 \leq s \leq 1, \; t \leq R.$$

Second, the policymaker chooses the optimal target $\hat{\theta}$ to maximize social welfare:

$$\max_{\hat{\theta}} \mathbb{E}_\theta[W(\theta,\hat{\theta};\sigma)] - C_{min}(\hat{\theta};\sigma),$$

$$\text{s.t.} \quad 0 \leq \hat{\theta} \leq \theta^*(\xi_0^*) = \frac{1}{R}.$$

In Section 3.2, we focus on the first step – that is, the cost minimization problem of achieving a target value of $\hat{\theta}$. As we will show, PPPs always incur a lower cost than do FPPs. It follows that the optimal intervention program must be a partial-participation program regardless of the optimal target θ. In Appendix A, we complete the analysis and show that the optimal target $\hat{\theta}$ depends on the long-term return R, the distribution function $F(\varepsilon)$ of investors' signals, the size of information friction σ, and the welfare cost τ per unit of transfer.

3.2 Cost Minimization

With regard to the *cost* of intervention, we assume that each dollar transferred from the policymaker to investors incurs a $\tau > 0$ unit of welfare loss. Here τ should not be interpreted as monetary value of the transfer, instead τ represents the policymaker's opportunity cost of forgoing other welfare-enhancing projects, and each dollar in the budget of the policymaker can be used to generates a welfare gain of τ.[11] Hence the welfare cost of offering intervention program (s,t) to each participating investor conditional on a fundamental realization θ is

$$c(\theta,s,t)=\begin{cases}-\tau t & \text{if } \theta>\theta^*(\xi^*(s,t)),\\ \tau s & \text{if } \theta\le\theta^*(\xi^*(s,t)).\end{cases} \tag{10}$$

This formulation speaks to the cost of implementing the program on the intensive margin. On the extensive margin, the mass of participating investors following a fundamental realization θ – which we denote as $P(\theta,s,t;\sigma)$ – depends (according to Proposition 2) on the type of program:

$$P(\theta,s,t;\sigma)=\begin{cases}1-F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right) & \text{if } t\le 0,\\ F\left(\frac{\eta^*(s,t)-\theta}{\sigma}\right)-F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right) & \text{if } 0<t\le s(R-1),\\ 0 & \text{if } t>s(R-1).\end{cases} \tag{11}$$

If $t\le 0$ then we have a full-participation program: all investors with a private signal that exceeds the run threshold participate in the intervention program. If $0<t\le s(R-1)$ then we have a partial-participation program, where only "pivotal investors" with private signals between the run and participation thresholds participate. If $t>s(R-1)$, then no investors participate in the intervention program.

The total welfare cost is simply the the cost per participant *multiplied by* the mass of participants:

$$C(\theta,s,t;\sigma)=c(\theta,s,t)P(\theta,s,t;\sigma). \tag{12}$$

As discussed in Section 3.1, the key to welfare maximization is finding the least costly intervention program that implements any fundamental threshold $\hat\theta$. Proposition 3 delivers a general result – on the cost efficiency of PPPs – that generates intuitions beyond those derived from our stylized benchmark model. Because a zero-participation program never implements a threshold that is strictly lower than the "laissez-faire" threshold $\theta^*(\xi_0^*)=1/R$, we need only compare PPPs and FPPs that implement the same fundamental threshold $\hat\theta$.

Proposition 3 *Given any $\sigma>0$, for any full-participation program (s,t) and any partial-participation program (s',t') that implement the same fundamental threshold $\hat\theta<1/R$,*

$$\mathbb{E}_\theta[C(\theta,s',t';\sigma)]<\mathbb{E}_\theta[C(\theta,s,t;\sigma)].$$

[11] We can think of a benevolent policymaker allocating her limited budget to maximize social welfare, in which case τ corresponds to the "shadow value" of her budget constraint.

This proposition states that the expected cost of any PPP is less than that of any FPP implementing the same fundamental threshold. Here we delineate the intuition underlying this result while leaving the proof to Appendix C. When the policymaker switches from a FPP (s, t) to a PPP (s', t'), the reduction in expected cost stems from both the extensive and intensive margins. On the extensive margin, the most optimistic investors – whose private signals are $x_i > \eta^*(s, t)$ – will no longer participate in the program. So if the bank fails then the policymaker pays subsidy s to fewer program participants, $P(\theta, s, t; \sigma)$. On the intensive margin, the subsidy and tax for each program participant changes from (s, t) to (s', t'). Note that the equilibrium run threshold ξ^* is the same under both programs, from which it follows that investors with $x_i \in [\xi^*, \eta^*(s', t'))$ would participate in either program. Among these investors, the marginal investor with signal $x_i = \xi^*$ is indifferent between (s, t) and (s', t') because his expected payoff is zero under both programs. All other participating investors, for whom $x_i \in (\xi^*, \eta^*(s', t'))$, strictly prefer the FPP. Because they estimate a higher probability of bank survival than does the marginal investor, they expect to pay more tax under the PPP. In other words, all these participating investors receives a lower expected transfer from the PPP than from the FPP. So from the policymaker's perspective, the PPP incurs a lower expected cost on the intensive margin. In sum: any PPP is less costly than any FPP because the former covers fewer participants and makes a lower expected transfer to each of them.

It is instructive to compare the expected cost of implementation of PPPs and FPPs graphically. Figure 3 (in Section 2.4) plots an investor's expected payoff as a function of his interim belief p_i, which is the probability of bank survival given i's private signal x_i. When the equilibrium fundamental threshold is $\hat{\theta}$, the interim belief can be expressed as

$$p_i = \mathbb{E}[\theta \geq \hat{\theta} \mid x_i].$$

So under any PPP and FPP that achieve the same target $\hat{\theta}$, investors with the same private signal have the same belief p_i and the distribution of investors' beliefs is the same in any state θ. Since each dollar of transfer from the policymaker to investors incurs a welfare cost τ, it follows that the most cost-efficient program features minimal expected transfers. Therefore, the policymaker should minimize investors' expected payoffs, represented by the blue lines in Figure 3. Thus the policymaker should reduce the slope of the straight line $\mathbb{E}[\pi_A(\theta, l)]$ while keeping its intercept with the horizontal axis at $\frac{1-s}{R-t-s} = \hat{\theta}$. If she does so, then the corresponding intervention program is associated with both a higher t and a higher s while the program's generosity (represented by the subsidy-to-tax ratio s/t) declines. When

s converges to 1, the cost of implementation converges to its lower bound.[12] We summarize these results in Corollary 1.

Corollary 1 *For two partial-participation programs* (s, t) *and* (s', t') *that implement the same fundamental threshold* $\hat{\theta}$*, the inequality* $\mathbb{E}_\theta[C(\theta, s', t'; \sigma)] < \mathbb{E}_\theta[C(\theta, s, t; \sigma)]$ *holds if and only if* $s' > s$*. When* $s \to 1$*, the expected cost converges to its lower bound:*

$$C_{min}(\hat{\theta}; \sigma) = \frac{\sigma\tau}{\bar{\theta} - \underline{\theta}} \int_{F^{-1}(\hat{\theta})}^{F^{-1}(1/R)} [1 - RF(y)] \, \mathrm{d}y > 0.$$

In the limit of negligible information frictions ($\sigma \to 0$), the mass of participants in any partial-participation program converges to zero for all realizations of θ *except* for the target threshold $\hat{\theta}$. Thus the expected cost of implementation converges to zero for all PPPs. In contrast, the mass of participants in any full-participation program is strictly positive when $\theta > \hat{\theta}$, and each participant receives a transfer of $-t \geq 0$ from the policymaker (or equivalently, pays a tax $t \leq 0$). So except for the government guarantee program with $t = 0$, all FPPs have a strictly positive expected cost of implementation. Although a government guarantee program and a partial-participation program both achieve zero expected cost, more investors participate in the former than in the latter. As a result, if investors' participation generates extra costs or creates distortions, a PPP will dominate a government guarantee program. For instance, to prevent bank runs with deposit insurance, policymakers need to set up deposit insurance funds. Even if no depositors file for claims, the administrative and opportunity costs of maintaining the fund can be enormous. Moreover, government guarantee programs can lead to moral hazard problems. In Section 4, we illustrate a major advantage of PPPs: their robustness with respect to moral hazard.

4 Interventions and Moral Hazard

It is widely accepted that the threat of bank runs can discipline moral hazard problems in financial institutions (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). For example, banks may shirk in credit assessment or make loans with a high default risk, and fund managers may engage in excessive risk taking because they do not bear the full cost when the downside of those risks becomes manifest. Investors can discipline financial institutions by the threat of early withdrawal. If a bank's portfolio is impaired, investors are likely to withdraw their funds early, which effectively increases the bank's incentive to maximize

[12] When $(s, t) = (1, R - 1)$, the program is a zero-participation program according to Proposition 2. So the infimum of the cost can be infinitely approached but never obtained.

the performance of its investment portfolio. Although government guarantees (e.g., deposit insurance) deter panic-based runs, they reduce investors' skin in the game and therefore limit the disciplinary effect of runs. In this section, we show that PPPs are more efficient than FPPs – including government guarantee programs – in the presence of moral hazard.

In order to incorporate moral hazard, we modify the benchmark model as follows. At $t = 0$, after the policymaker announces the intervention program but before the realization of θ, the bank decides whether (or not) to exert effort in screening and monitoring its long-term investments. We use $i^e \in \{0, 1\}$ to denote the bank's effort choice. If the bank exerts effort ($i^e = 1$), its long-term investments generate a return of R. However, if the bank shirks ($i^e = 0$), the quality of its investments deteriorate, and the return falls to $r \in (1, R)$. Therefore, an investor's payoff from staying and declining the policymaker's offered program ($a_i = 1_D$) is given by

$$\pi_D(\theta, l) = \begin{cases} Ri^e + r(1 - i^e) - 1 & \text{if } 1 - l \leq \theta, \\ -1 & \text{if } 1 - l > \theta; \end{cases}$$

his payoff from staying and accepting the offer ($a_i = 1_A$) is given by

$$\pi_A(\theta, l) = \begin{cases} Ri^e + r(1 - i^e) - 1 - t & \text{if } 1 - l \leq \theta, \\ -1 + s & \text{if } 1 - l > \theta. \end{cases}$$

An investor's payoff from early withdrawal ($a_i = 0$) is normalized to 0. If the bank shirks, investors receive a lower payoff from staying in the bank, which gives them an incentive to monitor the bank. For simplicity, we assume that the bank's choice of effort i^e is observable to investors so they can monitor the bank and condition their run decisions on the bank's effort choice.

The bank's payoff has two components. First, it incurs a cost $c^e > 0$ if it exerts effort but incurs no such cost if it shirks. Second, the bank receives a payoff $w > 0$ if it survives at $t = 2$ or a zero payoff if it defaults. We can therefore express the bank's payoff as

$$\pi^b(\theta, l) = -c^e i^e + w\mathbb{1}(1 - l \leq \theta).$$

In making its effort decision, the bank faces a trade-off. Shrinking saves the cost of effort, but it also induces more runs by investors and so increases the probability of default. We impose the following assumption so that exerting effort ($i^e = 1$) is the socially optimal action.

Assumption 1 *Shirking is inefficient,* $c^e < R - r$.

Hence the first-best scenario is one in which (a) the bank exerts effort and (b) all investors stay in the bank if and only if the fundamental $\theta \geq 0$. Everything else stays the same as in our benchmark model.

We showed in Section 3 that, if information frictions are nonnegligible ($\sigma > 0$), then PPPs incur strictly lower costs than do FPPs. Therefore, we shall next focus on the case of negligible information frictions ($\sigma \to 0$). In this case, government guarantee programs and PPPs both cost nothing to implement yet the latter are preferable because of their robustness to moral hazard.

The modified game with moral hazard can be solved via backward induction. Conditional on the bank's effort choice, investors follow a threshold investment strategy in equilibrium (as summarized in Proposition 2). Let $\theta_R = \frac{1-s}{R-t-s}$ be the fundamental threshold if the bank exerts effort, and let $\theta_r = \frac{1-s}{r-t-s}$ be the fundamental threshold if the bank shirks. In the limit of negligible information frictions, the probability of bank failure is proportional to the fundamental threshold (θ_R or θ_r depending on the bank's effort choice). Therefore, the bank is willing to exert effort if and only if the expected benefit from survival exceeds the cost of effort:

$$w\frac{\theta_r - \theta_R}{\bar{\theta} - \underline{\theta}} \geq c^e. \tag{13}$$

Since $\theta_r \leq 1/r$ and since $\theta_R \geq 0$ under any intervention program, it follows that the expected benefit of effort is bounded from above by w/r. We make the following assumption about the cost of effort such that investors' monitoring is effective.

Assumption 2 *Bank runs are sufficiently costly for the bank,* $c^e \leq \frac{w}{r(\bar{\theta}-\underline{\theta})}$.

If Assumption 2 is not valid, then the bank will always shirk regardless of any intervention program offered by the policymaker.

Recall from Proposition 2 that an intervention program with $t \leq 0$ is a FPP irrespective of the bank's effort choice. Because such programs provide free insurance against bank failures, they reduce investors' skin in the game and thus their incentive to monitor the bank. To be more explicit, consider a policymaker who seeks to achieve the first-best outcome with $i^e = 1$ and $\theta_R = 0$. In order to implement $\theta_R = 0$, a FPP must guarantee all losses due to bank failure (i.e., $s = 1$ and $t \leq 0$). Under such a program, however, investors have no incentive to monitor the bank. In other words: even if the bank shirks, investors' run decisions remain the same; that is, $\theta_r = \theta_R = 0$. So with a FPP, shirking is costless to the bank and so it will not exert effort ($i^e = 0$).

Now consider a partial-participation program that implements $\theta_R = 0$. By Proposition 2, this program will satisfy $s = 1$ and $0 < t \leq R - 1$. Although this PPP also guarantee all losses from bank failure, that guarantee is costly to the participating investors when the bank survives. When $t \leq r-1$, the cost of participating in the program is so low that, even if the bank shirks, the PPP remains viable and implements the threshold $\theta_r = 0$. Hence the program is still too generous to induce investor monitoring, and the bank shirks because the probability of default does not change. However, if $r-1 < t \leq R-1$, then the cost of participating in the program is so high that, when the bank shirks, the PPP becomes a ZPP. Therefore, investors follow the laissez-faire threshold $\theta_r = 1/r$ if the bank shirks. From the bank's perspective, shirking increases the default probability from 0 to $1/r$. Assumption 2 implies that the bank would rather exert effort and thereby benefit from the intervention program's stabilizing effect.

We summarize the foregoing analysis in our next proposition.[13]

Proposition 4 *When the information frictions are negligible* ($\sigma \to 0$)*,* (a) *no FPP achieves the first-best outcome and* (b) *any PPP* (s,t) *with* $s = 1$ *and* $r-1 < t < R-1$ *does achieve the first-best outcome.*

Under a FPP, the policymaker faces a trade-off between coordination efficiency and effort efficiency. On the one hand, insuring investors against losses from bank failure incentivizes them to stay in the bank. On the other hand, that guarantee reduces their incentive to monitor the bank. If the policymaker aims to eliminate coordination failure, then she must tolerate the bank's resulting moral hazard problem. In contrast, a PPP not only deters bank runs but also prevents banks from shirking. The subsidy incentivizes investors to stay in the bank while the tax restores their incentive to monitor the bank. So if a bank shirks then the tax discourages investors from participating in the program, which eliminates its stabilizing effect and thereby increases the likelihood of bank runs. Hence the threat of bank runs prevents banks from shirking. Therefore, a PPP with a well-designed tax-to-subsidy ratio preserves investors' disciplinary role in reducing the bank's moral hazard.

5 Extensions

In previous sections we have shown that partial-participation programs can reduce panic-based bank runs at a low fiscal cost and are also robust to moral hazard problems. Our goal here is to demonstrate the generality and robustness of our model. In Section 5.1, we start by discussing alternative applications of the model in financial markets that are susceptible to coordination failures. Then, in Section 5.2, we explore moral hazard problems of individual investors that are applicable to these alternative situations.

[13] The proof of the proposition follows directly from the foregoing analysis.

5.1 Market Freezes

Our model has a wide range of applications to circumstances under which a policymaker seeks to reduce coordination failures. As long as the policymaker can observe agents' actions and offer state-contingent programs, she can employ partial-participation programs to tackle coordination failures while preserving her fiscal budget and curbing moral hazard. Here we discuss the model's application to financial markets that freeze due to coordination failures among market participants.

It is widely agreed that coordination failures among financial institutions and investors contributed to the 2008 financial crisis.[14] Various asset markets suffered from runs and liquidity dry-up, and asset prices fell well below their fundamentals.[15] The credit market also froze, because banks hoard liquidity when anticipating a sluggish real economy.[16] To support liquidity in asset and credit markets, governments around the world provided substantial assistance – including granting access to cheap funding and acquiring illiquid assets – to financial institutions. In this way, governments made implicit and explicit guarantees on the financial system's behalf. Yet such large-scale intervention programs imposed a heavy fiscal burden, which led to an increase in sovereign risk.[17] These intervention programs also raised moral hazard concerns because they reduced the "skin in the game" of banks, a precursor of excessive risk taking and of shirking the assessment of credit risk.

We shall now discuss in more detail how freezes of the credit and asset markets can be incorporated into our framework as well as the implications for policies that aim to rejuvenate frozen markets. In the context of a *credit market* freeze, the strategic players are individual banks. More specifically, each bank i decides whether or not to lend to the real economy (a_i). If enough banks lend ($a_i = 1$), then the aggregate credit supply supports the real economy's growth, and all lending banks earn positive profits. But if enough banks abstain from lending ($a_i = 0$), then the real economy suffers from an aggregate credit crunch, and all lending banks incur losses. To boost the supply of credit, governments worldwide provided

[14] See Brunnermeier (2009) and Gorton (2010) for a review of the factors, which are reminiscent of bank runs, that contributed to that crisis.

[15] Among others, Gorton and Metrick (2012) and Covitz, Liang and Suarez (2013) document investor runs on the market for repos and asset-backed commercial paper. Bernardo and Welch (2004) model runs on financial markets in which investors pre-emptively sell assets because they fear a surge in future liquidation costs. Brunnermeier and Pedersen (2009) present a model of the feedback between asset market liquidity and traders' funding liquidity, which can result in a sudden dry-up of liquidity and in asset prices moving away from their fundamentals.

[16] Bebchuk and Goldstein (2011) model self-fulfilling credit market freezes when banks abstain from lending because they expect other banks to reduce their supply of credit contributing to a sluggish real economy.

[17] Over the period 2008–2014, accumulated gross financial sector assistance amounted to 8% of the eurozone's gross domestic product (Domingues Semeano et al., 2018). In some European countries, government rescues of financial institutions during the financial crisis ended up contributing to the subsequent sovereign debt crisis (Acharya, Drechsler and Schnabl, 2014).

cheap debt financing and also injected equity capital into banks. Our model reveals an advantage of equity injection relative to debt financing. Equity injection resembles a partial-participation program in our model. Because participating banks must share their lending profits with the government, the most optimistic banks would opt out of the equity injection program – reducing implementation costs. At the same time, all banks are aware of the offer; hence they anticipate a high aggregate credit supply and a prosperous economy, which gives them more incentive to lend.

In the context of *asset market* freeze, the strategic players are potential asset investors who individually decide whether to acquire the troubled assets (a_i). On the frozen asset market, if enough investors make new purchases or stick to their existing asset holdings ($a_i = 1$), then the asset market thaws and asset prices once again reflect the fundamentals. In this case, investors with $a_i = 1$ earn profits from their investment. Yet if enough investors stay out of the market or liquidate their existing holdings ($a_i = 0$), then the asset market remains frozen and so investors with $a_i = 1$ suffer losses. During the crisis, policymakers attempted to thaw frozen asset markets by establishing special credit facilities that accept these troubled assets as collateral to inject liquidity to market participants.[18] These special credit facilities resemble the *full*-participation programs in our model in that they increase the collateral value of troubled assets and benefit all financial institutions that participate in the market. We argue that policymakers can save implementation costs by switching to a *partial*-participation program, which can be implemented as a costly insurance that guarantees the asset's price. If an investor signs up for the program, he incurs a cost – making a monetary payment or conforming to stricter regulation – in exchange for an opportunity to sell the asset to the policymaker at a predetermined price. This boosts market confidence while making sure that the most optimistic investors opt out of the program to maintain a small size of intervention.

As demonstrated by our main model, full-participation programs can lead to moral hazard problems at the aggregate level: if a bank shirks, then all of its investors suffer from a lower payoff. However, moral hazard problems in the context of market freezes arise at the individual level and so affect only individual payoffs. On the credit market, for instance, FPPs reduce individual banks' "skin in the game" and can thereby lead to slack credit assessment. On the market for asset-backed securities and mortgage-backed securities, FPPs may result in moral hazard because they reduce the incentive of investors to monitor bor-

[18]For example, the US Federal Reserve – in response to the financial crisis – established the Term Asset-Backed Securities Loan Facility, the Commercial Paper Funding Facility, and the Term Securities Lending Facility.

rowers or mortgage servicers.[19] In Section 5.2, we show that partial-participation programs are robust to moral hazard at the individual level as well.

5.2 Individual Moral Hazard

Here we extend our benchmark model to incorporate individual moral hazard. We shall illustrate the robustness of partial-participation programs to moral hazard in the context of an asset market freeze. The extended model has two stages. The first stage is the same as the benchmark model *except* that investors do not receive their payoffs until the second stage. If the realized fundamental $\theta < 1-l$ in the first stage, then the market freezes and all investors who hold this type of asset suffer a net loss that we normalize to 1. If the first-stage realized fundamental $\theta \geq 1-l$, then the market is liquid, and each investor's payoff depends on his individual effort choice in the second stage. An investor who exerts effort to monitor the borrower incurs a cost of effort $c^e > 0$ and earns a profit $R-1$ with probability 1. However, an investor who shirks will suffer a loss of 1 with probability $\gamma \geq 0$ and will earn a profit $R-1$ with probability $1-\gamma$.

Just as in the benchmark model, we consider intervention programs characterized by a tax t and a subsidy s. An important friction here is that the policymaker can *not* contract on the effort choice of each individual investor; instead, her tax and subsidy payments can be based only on the gain and loss of each individual investor. In light of this friction, an intervention program stipulates that a participant must pay tax t if he profits from an investment and will receive a subsidy s if he suffers a loss. We make the following assumptions about the parameters involved.

Assumption 3 *The investment payoff has the following properties:*

(i) shirking is inefficient, $c^e < \gamma R$;

(ii) investing is ex ante efficient, $c^e < R-1$.

This assumption implies that the first-best scenario is that (a) all investors invest and exert effort if the fundamental $\theta \geq 0$ and (b) all investors choose not to invest otherwise. Assumption 3 also implies that, if the market is liquid ($\theta \geq 1-l$), a nonparticipating investor will exert effort for sure; however, an investor who participates in the intervention program (s,t) exerts effort if and only if his incentive compatibility (IC) constraint,

$$R - t - c^e \geq (1-\gamma)(R-t) + \gamma s, \tag{14}$$

[19] Kruger (2018) documents the moral hazard of servicers who underinvested in costly loss mitigation and therefore preferred loan foreclosure to loan modification.

is met. Therefore, to induce effort from participating investors, there is an upper bound on the sum of tax and subsidy:

$$t + s \leq R - \frac{c^e}{\gamma}. \tag{15}$$

We remark that $t + s$ represents the intervention program's distortion of the difference in the payoff from exerting effort versus shirking. If $t + s$ exceeds the threshold, then investors do not have enough skin in the game to exert effort, resulting in shirking. With a higher cost of effort c^e or with a lower probability γ of losses due to shirking, the incentive problem is more severe and so imposes a tighter constraint on the intervention program's generosity. We therefore make the following assumption to focus on the parameter region in which the IC constraint is binding.

Assumption 4 *The cost of effort is high,* $c^e > \gamma(R - 1)$.

It will become evident later that, in this parameter region, the policymaker faces a trade-off between inducing effort and reducing coordination failure. As in Section 4, we focus on the limit of negligible information friction. In this case, government guarantee programs and PPPs both cost nothing to implement yet the latter are preferable because of their robustness to individual moral hazard.

Government Guarantee As in the benchmark model, all investors will participate in a government guarantee program with $t = 0$ and $s > 0$. The expected payoff for an investor with interim belief p_i is therefore

$$\mathbb{E}[\pi_A(\theta, l) \mid p_i] = \begin{cases} p_i(R - c^e) + (1 - p_i)s - 1 & \text{if } s \leq R - \frac{c^e}{\gamma}, \\ p_i(1 - \gamma)R + (1 - p_i + p_i\gamma)s - 1 & \text{if } s > R - \frac{c^e}{\gamma}. \end{cases} \tag{16}$$

Our analysis of the benchmark model established that, in the unique equilibrium, the fundamental threshold above which the market is liquid equals the belief of the marginal investor who is indifferent between investing or not. If the policymaker prevents shirking by setting $s \leq R - c^e/\gamma$, then in equilibrium the fundamental threshold becomes

$$\theta^* = \frac{1 - s}{R - c^e - s}. \tag{17}$$

However, Assumption 4 implies that the most ambitious program with $s = R - c^e/\gamma < 1$ cannot insure against all investment losses. Therefore, programs aimed to prevent shirking cannot achieve the first-best outcome with $\theta^* = 0$.

If the policymaker tolerates shirking by setting $s > R - c^e/\gamma$, then the fundamental threshold becomes

$$\theta^* = \frac{1-s}{(1-\gamma)(R-s)}. \tag{18}$$

Indeed, the policymaker can eliminate all coordination failure and achieve $\theta^* = 0$ with a program with $s = 1$ that insures against any loss from investment. Yet in that case, the policymaker must tolerate the inefficiency due to the resulting moral hazard. In summary, within the framework of a government guarantee program, the policymaker faces a trade-off between inducing effort and encouraging investment. Next we show that, by implementing a partial-participation program, the policymaker can both encourage investment and induce effort from nearly all investors.

Partial-Participation Programs Now we look for a partial-participation program that can implement the first-best outcome. As in the main model, ensuring the effectiveness of a PPP requires that we set the tax and subsidy such that (a) the most optimistic investors choose $a_i = 1_D$ and (b) the pivotal investors, with interim beliefs around $1/R$, choose $a_i = 1_A$.

Investors who decide not to participate in the program ($a_i = 1_D$) endogenize all the payoffs from their investment. By Assumption 3(i), they always exert effort. In the case of individual-level moral hazard, we can see that the shirking option renders the intervention program more attractive. An optimistic investor, who believes that the market will certainly thaw ($p_i = 1$), will take action $a_i = 1_D$ if and only if

$$R - c^e - 1 > \max\{R - c^e - 1 - t, (1-\gamma)(R-t) + \gamma s - 1\}. \tag{19}$$

According to this inequality, an investor's payoff from $a_i = 1_D$ is higher than that from $a_i = 1_A$ regardless of whether he exert efforts or shirks. Hence condition (19) ensures that the option to shirk does not make the intervention program *too* attractive, and the most optimistic investors choose $a_i = 1_D$.

An investor with interim belief $p_i = 1/R$ is indifferent between $a_i = 1_D$ and $a_i = 0$, since both generate zero payoff. Under an intervention program, he strictly prefers $a_i = 1_A$ if and only if

$$\max\left\{\frac{1}{R}(R - c^e - t) + \left(1 - \frac{1}{R}\right)s - 1, \frac{1}{R}(1-\gamma)(R-t) + \left(1 - \frac{1}{R} + \frac{1}{R}\gamma\right)s - 1\right\} > 0. \tag{20}$$

This inequality guarantees that investors with interim beliefs $p_i = 1/R$ strictly prefers $a_i = 1_A$, and therefore, investors with beliefs slightly below $1/R$ will choose $a_i = 1_A$ regardless of effort choice. Absent an intervention program, they would forgo investing in the market

($a_i = 0$). Yet the PPP gives them more incentive to invest, which boosts aggregate investment in the market. All investors are aware of that increased aggregate investment and so become more optimistic about the market's prospects. Therefore, the mere announcement of PPP helps restore the liquidity of the asset market.

This extension echoes our main model in the sense that, as information friction σ approaches zero, so does the mass of investors who participate in the program. Recall that investors who decline to participate always exert effort. So even if participating investors shirk, the inefficiency is limited by their small mass. In contrast to government guarantee programs, partial-participation programs can achieve both goals: minimizing coordination failure and inducing effort.

Proposition 5 *Given Assumptions 3 and 4, if $\sigma \to 0$, then no government guarantee program can restore the first-best outcome. In contrast, the equilibrium outcome under a partial-participation program (s, t) with $s = 1$ and $\frac{c^e - \gamma R + \gamma}{1-\gamma} < t < R - 1$ converges to the first-best outcome. Furthermore, the ex ante cost of implementing such a partial-participation program also converges to zero.*

This proposition demonstrates the advantage of partial-participation programs over government guarantee programs when moral hazard problem is severe. The subsidy of an intervention program encourages investment in the first stage but deters effort input in the second stage. Under a government guarantee program, the policymaker faces the trade-off between first-stage investment efficiency and second-stage effort efficiency. Yet notwithstanding the moral hazard problem, a PPP still achieves the first-best outcome at zero cost. The advantage of PPPs with respect to individual-level moral hazard is that they involve only a small mass of participating investors. Although these investors shirk in the second stage, the effect of that shirking on social welfare is limited because the mass of these investors approaches zero as the information friction vanishes. In general: PPPs are preferable to FPPs, such as government guarantees, in the presence of inefficiencies that are proportional to the mass of participants.

6 Conclusions

This study analyzes a coordination game involving bank runs in the global games framework and proposes a novel type of intervention program that policymakers can use to reduce the likelihood of bank runs. The proposed intervention program screens and supports pivotal investors who receive medium signals. Due to strategic complementarities and amplification by higher-order beliefs, correctly incentivizing these pivotal investors has a significant effect on all investors. Compared with conventional government guarantee programs, the partial-participation programs proposed in this paper is not only less costly to implement but also more robust to moral hazard.

Although our partial-participation program has a wide range of applications in preventing coordination failures in financial systems, we point out its limitations as a concluding remark. First, a PPP requires the policymaker to observe investors' actions and then to condition her offer of the program on their run decisions. This is not always feasible. In the context of panic-based currency attack, for example, it can be hard to trace the identities of currency holders to whom the intervention could be offered. Second, the effectiveness of our proposed program relies on investors being rational. If the rationality of investors is bounded, then the amplification effect (via higher-order beliefs) will accordingly be limited.

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Appendices

A Optimal Choice of Target Threshold

In this section we investigate the determinants of the optimal choice of $\hat{\theta}$ when there is no moral hazard problem. First we calculate the marginal cost of reducing $\hat{\theta}$ by taking partial derivative of C_{min} with respect to $\hat{\theta}$:

$$-\frac{\mathrm{d}}{\mathrm{d}\hat{\theta}}C_{min}(\hat{\theta};\sigma)=\frac{\sigma\tau}{\bar{\theta}-\underline{\theta}}\frac{1-R\hat{\theta}}{f(F^{-1}(\hat{\theta}))}>0. \tag{A.1}$$

Next we write down the total expected payoff of the investors by taking expectation of (7),

$$\mathbb{E}_{\theta}[W(\theta,\hat{\theta};\sigma)]=\frac{R}{\bar{\theta}-\underline{\theta}}\int_{\hat{\theta}}^{\bar{\theta}}\left[1-F\left(\frac{\xi^{*}-\theta}{\sigma}\right)\right]\mathrm{d}\theta-\frac{1}{\bar{\theta}-\underline{\theta}}\int_{\underline{\theta}}^{\bar{\theta}}\left[1-F\left(\frac{\xi^{*}-\theta}{\sigma}\right)\right]\mathrm{d}\theta. \tag{A.2}$$

Taking derivative with respect to $\hat{\theta}$, we have

$$\frac{\mathrm{d}}{\mathrm{d}\hat{\theta}}\mathbb{E}_{\theta}[W(\theta,\hat{\theta};\sigma)]=-\frac{R(1-\hat{\theta})}{\bar{\theta}-\underline{\theta}}-\frac{\mathrm{d}\xi^{*}}{\mathrm{d}\hat{\theta}}\frac{1}{\sigma(\bar{\theta}-\underline{\theta})}\left[R\int_{\hat{\theta}}^{\bar{\theta}}f\left(\frac{\xi^{*}-\theta}{\sigma}\right)\mathrm{d}\theta-\int_{\underline{\theta}}^{\bar{\theta}}f\left(\frac{\xi^{*}-\theta}{\sigma}\right)\mathrm{d}\theta\right], \tag{A.3}$$

$$=-\frac{R(1-\hat{\theta})}{\bar{\theta}-\underline{\theta}}-\frac{\mathrm{d}\xi^{*}}{\mathrm{d}\hat{\theta}}\frac{R\hat{\theta}-1}{\bar{\theta}-\underline{\theta}}, \tag{A.4}$$

$$=-\frac{R-1}{\bar{\theta}-\underline{\theta}}+\frac{\sigma(1-R\hat{\theta})}{(\bar{\theta}-\underline{\theta})f(F^{-1}(\hat{\theta}))}. \tag{A.5}$$

When $\hat{\theta}>0$, the total marginal increase in social welfare of reducing $\hat{\theta}$ is

$$-\frac{\mathrm{d}}{\mathrm{d}\hat{\theta}}\mathbb{E}_{\theta}[W(\theta,\hat{\theta};\sigma)]+\frac{\mathrm{d}}{\mathrm{d}\hat{\theta}}C_{min}(\hat{\theta};\sigma)=\frac{R-1}{\bar{\theta}-\underline{\theta}}-(1+\tau)\frac{\sigma(1-R\hat{\theta})}{(\bar{\theta}-\underline{\theta})f(F^{-1}(\hat{\theta}))}. \tag{A.6}$$

Although it is difficult to discipline the distribution function$F(\varepsilon)$, we can discuss the relationship between the optimal $\hat{\theta}$ and the other parameters. First, notice that when there exist positive information frictions ($\sigma>0$), the optimal $\hat{\theta}$ can be greater than zero – the first best fundamental threshold when there is no information friction. This is because information frictions can lead to imperfect coordination: a positive mass of investors stay in the bank when θ falls below the fundamental threshold. When $\hat{\theta}$ decreases, the run threshold ξ^{*} becomes relatively smaller compared to $\hat{\theta}$, and this makes the imperfect coordination problem more severe and more costly for the policy maker. Therefore, if there is an interior optimal $\hat{\theta} > 0$, it must increase in σ. Second, higher social cost of transfer τ implies a higher cost of imperfect coordination, as the policymaker needs to compensate the losses of the investors who suffer bank failure. Hence, an interior optimal $\hat{\theta}$ must increase with τ. Third, a higher gross return R implies a larger benefit of encouraging investment. In addition, a higher gross return R gives investors stronger incentives to invest outside of the optimal partial-participation program; it reduces the mass of participants and lowers the cost of imperfect coordination. Since both effects work in the same direction, an interior optimal $\hat{\theta}$ decreases with R. In summary, the policy maker will choose a more aggressive target fundamental (lower $\hat{\theta}$) when the gross return is higher or the information friction and the cost of transfer is lower. When $\sigma \to 0$, the optimal $\hat{\theta}$ converges to the first-best fundamental threshold $\theta^{FB} = 0$.

B General Payoff Structure

In this appendix, we follow the set-ups of the symmetric binary-action global games given by Morris and Shin (2003) and allow for general monotonic payoff functions.

As in the benchmark model presented in Section 2, an investor's payoff from withdrawing early ($a_i = 0$) is normalized to 0. However, an investor's payoff from staying ($a_i = 1$) is now modified to be a continuous function $\pi(x_i, l)$, which is weakly increasing in the private signal x_i and the total amount of funding that stays in the bank $l = \int_0^1 a_i \, \mathrm{d}i$.[20] The fundamental θ is distributed uniformly on $[\underline{\theta}, \bar{\theta}]$, where $\underline{\theta} < -\sigma$ and $\bar{\theta} > 1 + \sigma$. The private signal received by investor i is $x_i = \theta + \sigma\varepsilon_i$, where the ε_i are i.i.d. with density function $f(\varepsilon)$ and distribution function $F(\varepsilon)$ with support $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

We consider only linear intervention programs, since they are realistic and easy to implement. In principle, we could also consider nonlinear intervention programs; yet as we shall demonstrate, linear intervention programs already deliver most of the intuitions. An intervention program (s, t) consists of two parts: a lump-sum subsidy $s \geq 0$ and a proportional tax $t \in [0, 1]$. An investor who accepts the offer receives the lump-sum subsidy s and pays the proportional tax t once the coordination outcome is realized. His payoff from accepting the offer is[21]

$$\pi_A(x_i, l) = (1 - t)\pi_D(x_i, l) + s, \tag{B.1}$$

[20] To simplify the demonstration, we assume that the payoff is a function of the private signal, not of the fundamentals; our results continue to hold under the alternative payoff structure $\pi(\theta, l)$. See Morris and Shin (2003) for the discussion of the two approaches.

[21] One might notice that, when $\pi(x, l) < 0$, investors end up paying a "negative tax". Let $\underline{\pi} = \pi\big(\underline{\theta} - \frac{1}{2}\sigma, 0\big)$ be the lower bound of the payoff. Then the intervention program can be implemented by providing a positive subsidy $s - t\underline{\pi}$ and imposing a proportional tax t on the positive tax base $\pi(x, l) - \underline{\pi}$.

where $\pi_D(x_i, l) = \pi(x_i, l)$ is his payoff from declining the offer. Investors who receive low private signals anticipate a low realization of the fundamental θ and a low funding condition l, so they are pessimistic about their payoffs from staying in the bank. Hence they expect that the tax they pay will be low and so are more willing (than are optimistic investors) to accept the offer. Recall from our main model that partial-participation programs do not appeal to the most optimistic agents, which reduces the implementation costs and reduces moral hazard. The proportional tax t reflects this feature and helps target agents who receive medium signals.

We adopt several of the literature's standard assumptions regarding the payoff function.

Assumption B.1 *The payoff function $\pi(x_i, l)$ satisfies the following properties.*

(i) (Monotonicity) The payoff function $\pi(x_i, l)$ is weakly increasing in x_i and weakly increasing in l.

(ii) (Strict Laplacian State Monotonicity) $\int_0^1 \pi(x_i, l)\,\mathrm{d}l$ is strictly increasing in x_i.

(iii) (Limit Dominance) There exist $\theta_0, \theta_1 \in \left(\underline{\theta} + \frac{1}{2}\sigma, \bar{\theta} - \frac{1}{2}\sigma\right)$ such that

$$\pi(x_i, 1) < 0 \quad \text{for all } x_i < \theta_0, \tag{B.2}$$

$$\pi(x_i, 0) > 0 \quad \text{for all } x_i > \theta_1. \tag{B.3}$$

(iv) (Continuity) $\int_0^1 g(l)\pi(x_i, l)\,\mathrm{d}l$ is continuous in x_i for any density function g.

Part (i) of Assumption B.1 gives the strategic complementarities among investors: an investor's payoff from staying in the bank increases when the total funding staying the bank l increases or when his private signal x_i increases indicating a higher fundamental θ. Note that the payoff function need not be strictly increasing or continuous. For example, the payoff function in our benchmark model in Section 2 is a step function. The role of part (ii) is to ensure that the equilibrium is unique when it exists, with or without the intervention program. Part (iii) ensures the existence of two dominance regions – so that we can undertake the iterated deletion of dominated strategies from both sides. Part (iv) governs the integration of the payoff function so that there always exists an equilibrium.

The equilibrium without intervention is characterized in the following proposition. The coordination outcome in this case serves as a benchmark to highlight the effect of intervention programs.

Proposition B.1 *Without intervention ($s = t = 0$), if information frictions σ are small enough then there is a unique equilibrium whereby each investor stays in the bank if and only if his private signal $x_i \geq \xi_0^*$. The signal threshold ξ_0^* is given by*

$$\int_0^1 \pi(\xi_0^*, l)\,\mathrm{d}l = 0.$$

Let us compare the coordination results characterized in Proposition B.1 with the first-best outcome. The first-best scenario is that all investors follow the same cut-off strategy θ_0, which is the upper bound for the lower dominance region. By Assumption B.1, the coordination outcome $\xi_0^* > \theta_0$ unless $\pi(\theta_0, l) = 0$ for any $l \in [0, 1]$. In other words: if the realized fundamental $\theta \in (\theta_0, \xi_0^*)$, then a panic-based bank run will arise. Hence the goal of intervention is to reduce the coordination threshold from ξ_0^* to a value that is as close to θ_0 as possible.

Next we analyze the equilibrium with an intervention program (s, t). We focus on partial-participation programs defined as follows and demonstrate the zero cost of implementation in the limit of negligible information friction.

Definition B.1 *An intervention program (s, t) is a partial-participation program with target $\xi^* \in (\theta_0, \xi_0^*)$ if and only if it satisfies these three conditions:*

(i) (Intervention Target) $\int_0^1 \pi(\xi^*, l)\,\mathrm{d}l = -s/(1-t)$;

(ii) (Optimism Exclusion) $\pi(\xi^*, 1) > s/t$;

(iii) (Lower Dominance Region) $\pi(\underline{\theta}, 1) < -s/(1-t)$.

Denoting by $G(\sigma; s, t)$ the coordination game with information friction σ and intervention program (s, t), we can prove our next proposition.

Proposition B.2 *Suppose we have a partial-participation program (s, t) with target $\xi^* \in (\theta_0, \xi_0^*)$. Then, in the unique Bayesian Nash equilibrium of the coordination game $G(\sigma; s, t)$,*

$$a_i(x_i) = \begin{cases} 1_D & \text{if } x_i \geq \eta^*(\sigma), \\ 1_A & \text{if } \xi^* \leq x_i < \eta^*(\sigma), \\ 0 & \text{if } x_i < \xi^*. \end{cases}$$

As $\sigma \to 0$, $\eta^*(\sigma)$ converges to ξ^*.

Proposition B.2 gives the conditions under which there exist PPPs that reduce the run threshold to ξ^*. Just as in the main model with binary payoffs, in the limit of vanishing information frictions, we see that the ex ante expected mass of participants approaches zero, which implies a zero implementation cost. The only remaining question is whether there exists a PPP that can restore the first-best scenario (i.e., $\xi^* = \theta_0$). That question is answered by the following proposition.

Proposition B.3 *If $\int_0^1 \pi(\theta_0, l)\,\mathrm{d}l \geq \pi(\underline{\theta}, 1)$. Then, for any $\xi^* \in (\theta_0, \xi_0^*)$, there exists a partial-participation program with target ξ^*.*

In the presence of an intervention program, all investors become more optimistic about their investment payoff. It follows that the lower dominance region, where agents prefer to run on the bank even if $l = 1$, shrinks. The condition stipulated in Proposition B.3 guarantees that the lower dominance region still exists even when there is an intervention program. If that condition is violated, then there could be multiple equilibria when targeting ξ^* close to θ_0. However, if we follow the equilibrium refinements proposed in Goldstein and Pauzner (2005), then we can select the equilibrium described in Proposition B.2 even without the lower dominance region. So following those refinements, there always exists a partial-participation program that restores the first-best scenario. Moreover, the lower dominance region may disappear because we limit our attention to programs with linear transfers. Linear transfer schedules typically pay out high subsidies when the fundamentals are low. If the policymaker reduces subsidies in the case of extremely low realizations of the fundamentals (i.e., when $\theta < \theta_0$) or if she increases the convexity of the tax schedule appropriately, then both the left dominance region and the equilibrium's uniqueness can be recovered. In any case, there always exists an intervention program that can restore the first-best scenario.

C Proofs

Proof of Proposition 1. We follow the proof of Proposition 2.1 in Morris and Shin (2003). Here we outline the proof using iterated deletion of dominated strategies.

Let $p_i = p(x_i; k)$ denote the interim belief of an investor i who receives private signal x_i when all other investors follow a threshold strategy k as defined in (5). We first prove that a strategy survives n rounds of iterated deletion of dominated strategies if and only if

$$a_i(x_i) = \begin{cases} 0 & \text{if } x_i < \underline{\xi}_n, \\ 1 & \text{if } x_i \geq \bar{\xi}_n, \end{cases} \tag{C.1}$$

where $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ satisfies

$$-\infty = \underline{\xi}_0 < \underline{\xi}_1 \leq \cdots \leq \underline{\xi}_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \tag{C.2}$$

This result can be proved by induction. Let $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = +\infty$, so that the first round of deletion starts with the full set of strategies. Suppose round $n \in \mathbb{N}$ of deletion has been completed. In round $n+1$, the best scenario for an investor who wants to stay in the bank ($a_i = 1$) is that all other investors follow a threshold strategy $\underline{\xi}_n$, which minimizes total early withdrawal $1 - l$. So for any x_i such that $p(x_i; \underline{\xi}_n) < 1/R$, early withdrawing strictly dominates staying. Similarly, the best scenario for an investor who wants to withdraw early ($a_i = 0$) is that all other investors follow a threshold strategy $\bar{\xi}_n$. As a result, for x_i such that $p(x_i; \bar{\xi}_n) > 1/R$ we have that staying strictly dominates early withdrawing.

Because $p(x_i; k)$ is non-decreasing in x_i, the strategy profiles that survive iterated deletion of dominated strategies can be summarized in the form of (C.1); in those expressions, given $(\underline{\xi}_n, \bar{\xi}_n)$, $(\underline{\xi}_{n+1}, \bar{\xi}_{n+1})$ are defined inductively as

$$\underline{\xi}_{n+1} = \inf\left\{x_i : p(x_i; \underline{\xi}_n) \geq \frac{1}{R}\right\}, \tag{C.3}$$

$$\bar{\xi}_{n+1} = \sup\left\{x_i : p(x_i; \bar{\xi}_n) \leq \frac{1}{R}\right\}. \tag{C.4}$$

The monotonicity of $p(x_i; k)$ guarantees that $\underline{\xi}_{n+1} \leq \bar{\xi}_{n+1}$ provided $\underline{\xi}_n \leq \bar{\xi}_n$. Note that our assumption about the dominance regions implies the inequalities $\underline{\xi}_1 > -\infty$ and $\bar{\xi}_1 < +\infty$. Therefore, $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ is a well-defined sequence of real couples that satisfy (C.2).

Since $\{\underline{\xi}_n\}_{n=1}^{\infty}$ and $\{\bar{\xi}_n\}_{n=1}^{\infty}$ are each a monotonic and bounded sequence, it follows that – as $n \to \infty$ – (a) they must converge to two finite numbers $\underline{\xi}$ and $\bar{\xi}$ and (b) $\underline{\xi} \leq \bar{\xi}$. Equations (C.3) and (C.4) then imply that $p(\underline{\xi}; \underline{\xi}) \geq 1/R$ and $p(\bar{\xi}; \bar{\xi}) \leq 1/R$. We remark that

$$p(\xi; \xi) = F\left(\frac{\xi - \theta^*(\xi)}{\sigma}\right) = \theta^*(\xi) \tag{C.5}$$

is strictly increasing in ξ. Hence $\underline{\xi} = \bar{\xi}$ must be the unique solution to $\theta^*(\xi) = \frac{1}{R}$, which is

$$\xi_0^* = \frac{1}{R} + \sigma F^{-1}\left(\frac{1}{R}\right). \tag{C.6}$$

Because there is only one strategy that survives the iterated deletion of dominated strategies, the equilibrium of the game is unique and the associated equilibrium strategy is the threshold strategy ξ_0^*. ■

Proof of Proposition 2. Frankel, Morris and Pauzner (2003) prove existence, uniqueness, and monotonicity in multi-action global games. Since our set-up does not satisfy the continuity assumption, we provide our own proof here.

In case (i), $a_i = 1_D$ is dominated by $a_i = 1_A$ and so all investors who choose to stay will accept the offer. We can therefore set the payoff from staying at $\pi(\theta, l) = \pi_A(\theta, l)$ and directly apply Proposition 1. In case (iii), $a_i = 1_A$ is similarly dominated by $a_i = 1_D$ and so the offer is never accepted by any investor. Hence the equilibrium is exactly the same as in Proposition 1.

In case (ii), the optimal strategy of investor i conditional on his interim belief p_i is

$$a_i = \begin{cases} 1_D & \text{if } p_i > p_2^*, \\ 1_A & \text{if } p_1^* < p_i \le p_2^*, \\ 0 & \text{if } p_i \le p_1^*; \end{cases}$$

here $p_1^* = \frac{1-s}{R-t-s}$ and $p_2^* = \frac{s}{t+s}$ are two threshold beliefs that satisfy $0 \le p_1^* < p_2^* \le 1$. Note that the coordination outcome (i.e., the bank's survival) depends only on the aggregate early withdrawal $1 - l$; whether investors accept or decline the offer has no direct effect on the bank's survival. Therefore, we can define a sequence $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ such that a strategy survives n rounds of iterated deletion of dominated strategies if and only if

$$a_i = 0 \quad \text{if } x_i < \underline{\xi}_n, \tag{C.7}$$

$$a_i \in \{1_D, 1_A\} \quad \text{if } x_i \ge \bar{\xi}_n, \tag{C.8}$$

where the recursive expression for $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ is

$$\underline{\xi}_{n+1} = \inf\{x : p(x; \underline{\xi}_n) \ge p_1^*\}, \tag{C.9}$$

$$\bar{\xi}_{n+1} = \sup\{x : p(x; \bar{\xi}_n) \le p_1^*\}. \tag{C.10}$$

Applying the same techniques used in the proof of Proposition 1, we can see that the limit of the two threshold sequences converge to

$$\xi^*(s,t) = p_1^* + \sigma F^{-1}(p_1^*), \tag{C.11}$$

which is the run threshold in the unique Bayesian Nash equilibrium of the global game. The associated participation threshold η is the solution to

$$p(\eta; \xi^*(s,t)) = p_2^*. \tag{C.12}$$

Solving (C.12) yields

$$\eta^*(s,t) = p_1^* + \sigma F^{-1}(p_2^*). \tag{C.13}$$

Plugging in the expressions for the two threshold beliefs $p_1^* = \frac{1-s}{R-t-s}$ and $p_2^* = \frac{s}{t+s}$ completes the proof. ■

Proof of Proposition 3. Consider a full-participation program (s,t) and a partial-participation program (s',t') that implement the same fundamental threshold $\hat{\theta}$. The expected cost of the FPP (s,t) is

$$\mathbb{E}_\theta[C(\theta,s,t;\sigma)] = \frac{\tau s}{\bar{\theta}-\underline{\theta}} \int_{\underline{\theta}}^{\hat{\theta}} \left[1 - F\left(\frac{\xi^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta - \frac{\tau t}{\bar{\theta}-\underline{\theta}} \int_{\hat{\theta}}^{\bar{\theta}} \left[1 - F\left(\frac{\xi^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta.$$

The expected cost of the PPP (s',t') is

$$\begin{aligned}
\mathbb{E}_\theta[C(\theta,s',t';\sigma)] = {} & \frac{\tau s'}{\bar{\theta}-\underline{\theta}} \int_{\underline{\theta}}^{\hat{\theta}} \left[F\left(\frac{\eta^*-\theta}{\sigma}\right) - F\left(\frac{\xi^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta \\
& - \frac{\tau t'}{\bar{\theta}-\underline{\theta}} \int_{\hat{\theta}}^{\bar{\theta}} \left[F\left(\frac{\eta^*-\theta}{\sigma}\right) - F\left(\frac{\xi^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta,
\end{aligned}$$

where ξ^* and η^* are (respectively) the investment and participation thresholds given in Proposition 2: $\xi^* = \hat{\theta} + \sigma F^{-1}(\hat{\theta})$; and $\eta^*(s',t') = \hat{\theta} + \sigma F^{-1}\left(\frac{s'}{s'+t'}\right)$. To simplify the notation, we shall no longer indicate the dependence of ξ^* and η^* on (s',t'). The difference between the cost of FPP (s,t) and the cost of PPP (s',t') can be decomposed into two parts,

$$\mathbb{E}_\theta[C(\theta,s,t)] - \mathbb{E}_\theta[C(\theta,s',t')] = \frac{\tau}{\bar{\theta}-\underline{\theta}}(\Delta_1 + \Delta_2),$$

where

$$\begin{aligned}
\Delta_1 &= s \int_{\underline{\theta}}^{\hat{\theta}} \left[1 - F\left(\frac{\eta^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta - t \int_{\hat{\theta}}^{\bar{\theta}} \left[1 - F\left(\frac{\eta^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta, \\
\Delta_2 &= (s-s') \int_{\underline{\theta}}^{\hat{\theta}} \left[F\left(\frac{\eta^*-\theta}{\sigma}\right) - F\left(\frac{\xi^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta \\
&\quad - (t-t') \int_{\hat{\theta}}^{\bar{\theta}} \left[F\left(\frac{\eta^*-\theta}{\sigma}\right) - F\left(\frac{\xi^*-\theta}{\sigma}\right)\right] \mathrm{d}\theta.
\end{aligned}$$

Here Δ_1 and Δ_2 represent the cost difference on the extensive and intensive margins, respectively. Next we must determine the signs of Δ_1 and Δ_2. Because $s > 0$ and $t \le 0$ in a FPP,

$\Delta_1 > 0$. To simplify Δ_2, observe that

$$\int_{\alpha}^{\beta}\left[F\left(\frac{\eta^*-\theta}{\sigma}\right)-F\left(\frac{\xi^*-\theta}{\sigma}\right)\right]\mathrm{d}\theta=\int_{\xi^*}^{\eta^*}\left[F\left(\frac{x-\alpha}{\sigma}\right)-F\left(\frac{x-\beta}{\sigma}\right)\right]\mathrm{d}x.$$

Therefore,

$$\begin{aligned}\Delta_2 &= (s-s')\int_{\xi^*}^{\eta^*}\left[1-F\left(\frac{x-\hat{\theta}}{\sigma}\right)\right]\mathrm{d}x-(t-t')\int_{\xi^*}^{\eta^*}F\left(\frac{x-\hat{\theta}}{\sigma}\right)\mathrm{d}x\\ &= (s'-s)\int_{\xi^*}^{\eta^*}\left[\left(1+\frac{t'-t}{s'-s}\right)F\left(\frac{x-\hat{\theta}}{\sigma}\right)-1\right]\mathrm{d}x.\end{aligned}$$

Since (s,t) and (s',t') implement the same fundamental threshold $\hat{\theta}$, it follows that $(1-s)=\hat{\theta}(R-t-s)$ and $(1-s')=\hat{\theta}(R-t'-s')$. Taking the difference of these two equations yields

$$\frac{t'-t}{s'-s}=\frac{1-\hat{\theta}}{\hat{\theta}}.$$

Plugging this equality into our expression for Δ_2, we obtain

$$\begin{aligned}\Delta_2 &= (s'-s)\int_{\xi^*}^{\eta^*}\left[\frac{1}{\hat{\theta}}F\left(\frac{x-\hat{\theta}}{\sigma}\right)-1\right]\mathrm{d}x\\ &> (s'-s)\int_{\xi^*}^{\eta^*}\left[\frac{1}{\hat{\theta}}F\left(\frac{\xi^*-\hat{\theta}}{\sigma}\right)-1\right]\mathrm{d}x=0.\end{aligned}$$

Both Δ_1 and Δ_2 are positive; therefore, $\mathbb{E}_\theta[C(\theta,s,t;\sigma)]>\mathbb{E}_\theta[C(\theta,s',t';\sigma)]$ for any FPP (s,t) and any PPP (s',t') that implement the same fundamental threshold. ■

Proof of Corollary 1. We have shown that conditional on a target fundamental threshold $\hat{\theta}=\frac{1-s}{R-t-s}$, the expected cost of implementation converges to the lower bound when s and s/t converge to 1 and $1/(R-1)$ respectively. In the limit,

$$\begin{aligned}\xi^* &= \hat{\theta}+\sigma F^{-1}(\hat{\theta}),\\ \eta^* &= \hat{\theta}+\sigma F^{-1}\left(\frac{1}{R}\right).\end{aligned}$$

We take expectation of (12) over all realizations of θ to compute the minimum expected cost

$$
\begin{aligned}
C_{min}(\hat{\theta};\sigma) =& \lim_{s\to 1} \ \mathbb{E}_\theta[C(\hat{\theta}, s, R-s-\frac{1-s}{\hat{\theta}};\sigma)],\\
=&\frac{\tau}{\bar{\theta}-\underline{\theta}}\int_{\xi^*}^{\eta^*}\left[1-F\left(\frac{x-\hat{\theta}}{\sigma}\right)\right]\mathrm{d}x - \frac{\tau(R-1)}{\bar{\theta}-\underline{\theta}}\int_{\xi^*}^{\eta^*}F\left(\frac{x-\hat{\theta}}{\sigma}\right)\mathrm{d}x,\\
=&\frac{\sigma\tau}{\bar{\theta}-\underline{\theta}}\int_{F^{-1}(\hat{\theta})}^{F^{-1}(1/R)}[1-RF(y)]\,\mathrm{d}y > 0.
\end{aligned}
$$

■

Proof of Proposition 4. A proof has been provided in Section 4. ■

Proof of Proposition 5. The government guarantee program that achieves a target fundamental threshold $\hat{\theta}=0$ is $(s,t)=(1,0)$, which guarantees any loss from failed investment. However, such program violates the incentive compatibility constraint (condition (14)) and so all participating investors would shirk, incurring welfare loss.

Next we show that a partial-participation program (s,t) with $s=1$ and $t\in\left(\frac{c^e-\gamma R+\gamma}{1-\gamma}, R-1\right)$ can restore the first-best outcome when $\sigma\to 0$. Under such program, investor i's optimal strategy – conditional on his interim belief – is

$$
a_i = \begin{cases} 1_D & \text{if } \ p_i \geq p_2^*, \\ 1_A & \text{if } \ p_1^* \leq p_i < p_2^*, \\ 0 & \text{if } \ p_i < p_1^*; \end{cases}
$$

here

$$
\begin{aligned}
p_2^* &= \frac{s}{\gamma R + (t+s)(1-\gamma) - c^e} < 1,\\
p_1^* &= \frac{1-s}{(1-\gamma)(R-t-s)} = 0.
\end{aligned}
$$

Following the proof of Proposition 2(ii), we can establish the existence of a unique equilibrium in which any investor i adopts the following strategy:

$$
a_i = \begin{cases} 1_D & \text{if } \ x_i \geq \eta^*, \\ 1_A & \text{if } \ \xi^* \leq x_i < \eta^*, \\ 0 & \text{if } \ x_i < \xi^*; \end{cases}
$$

here

$$\xi^* = p_1^* + \sigma F^{-1}(p_1^*),$$
$$\eta^* = p_1^* + \sigma F^{-1}(p_2^*).$$

When $\sigma \to 0$, both ξ^* and η^* converge to $p_1^* = 0$. Hence this PPP achieves the first-best outcome: for any fundamental $\theta > 0$, all investors take action $a_i = 1_D$ and exert effort; for any fundamental $\theta < 0$, all investors take action $a_i = 0$. The mass of participants in the program is zero (except when $\theta = 0$); hence for any continuous distribution of the fundamental, the program's ex ante cost of implementation converges to zero. ■

Proof of Proposition B.1. Consider an investor who receives private signal x_i and knows that all other investors follow threshold strategy k – that is, run on the bank if and only if the private signal $x_i < k$. Then the expected payoff from staying in the bank is

$$U(k, x_i) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x_i - \theta}{\sigma}\right) \pi\left(x_i, 1 - F\left(\frac{k - \theta}{\sigma}\right)\right) \mathrm{d}\theta.$$

Note that $U(k, x_i)$ weakly decreases with k and weakly increases with x_i. That is: an investor has higher expected payoff from staying in the bank if (a) others are more willing to stay in the bank or (b) the investor receives a high signal indicating a high fundamental θ.

Next we prove the uniqueness of equilibrium via iterated deletion of dominated strategies. The strategy profile of an investor amounts to his action as a function of his private signal. We denote this profile by $a_i(x_i)\colon \mathbb{R} \to \{0, 1\}$. We will prove that a strategy survives n rounds of iterated deletion of dominated strategies if and only if

$$a_i(x_i) = \begin{cases} 0 & \text{if } x_i < \underline{\xi}_n, \\ 1 & \text{if } x_i \geq \bar{\xi}_n. \end{cases} \tag{C.14}$$

Here $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ satisfies

$$-\infty = \underline{\xi}_0 < \underline{\xi}_1 \leq \cdots \leq \underline{\xi}_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \tag{C.15}$$

This result can be proved by induction. Let the starting nodes be $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = +\infty$ so that there are no restrictions on investors' strategies. Consider all strategy profiles that survive $n \in \mathbb{N}$ rounds of deletion. In round $n+1$, the most *optimistic* belief for an investor is that all other investors follow a threshold strategy $\underline{\xi}_n$. So for any x_i such that $U(\underline{\xi}_n, x_i) < 0$, we have that staying ($a_i(x_i) = 1$) is strictly dominated by early withdrawing ($a_i(x_i) = 0$).

The most *pessimistic* belief for an investor is, analogously, that all other investors follow a threshold strategy $\bar{\xi}_n$. So for x_i such that $U(\bar{\xi}_n, x_i) > 0$, any strategy profile with $a_i(x_i) = 0$ is strictly dominated by $a_i(x_i) = 1$.

Because $U(k, x_i)$ is non-decreasing in x_i, a strategy profile that survives deletion of dominated strategies must satisfy the restrictions given in (C.14) with $(\underline{\xi}_{n+1}, \bar{\xi}_{n+1})$ defined inductively as

$$\underline{\xi}_{n+1} = \inf\{x : U(\underline{\xi}_n, x_i) \geq 0\}, \tag{C.16}$$

$$\bar{\xi}_{n+1} = \sup\{x : U(\bar{\xi}_n, x_i) \leq 0\}. \tag{C.17}$$

The monotonicity of $U(k, x_i)$ guarantees that $\underline{\xi}_{n+1} \leq \bar{\xi}_{n+1}$. Note that the Limit Dominance assumption (part (iii) of Assumption B.1) implies $U(-\infty, x_i) < 0$ for $x_i < \theta_0$ and $U(+\infty, x_i) > 0$ for $x_i > \theta_1$, and so $\underline{\xi}_1 > -\infty$ and $\bar{\xi}_1 < +\infty$. Hence $\{(\underline{\xi}_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ is a well-defined sequence of real couples that satisfy (C.15).

We have proved that each of $\{\underline{\xi}_n\}_{n=1}^{\infty}$ and $\{\bar{\xi}_n\}_{n=1}^{\infty}$ is a monotonic and bounded sequence. Hence they converge to (respectively) the finite numbers $\underline{\xi}$ and $\bar{\xi}$ when $n \to \infty$. The definitions (C.16) and (C.17) imply that $U(\underline{\xi}, \underline{\xi}) \geq 0$ and $U(\bar{\xi}, \bar{\xi}) \leq 0$.

For any $y \in [\theta_0, \theta_1]$, we have that

$$U(y, y) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{y - \theta}{\sigma}\right) \pi\left(y, F\left(\frac{y - \theta}{\sigma}\right)\right) \mathrm{d}\theta = \int_0^1 \pi(y, l) \, \mathrm{d}l,$$

which strictly increases with y by the Strict Laplacian State Monotonicity (part (ii) of Assumption B.1). Therefore, $U(\underline{\xi}, \underline{\xi}) \geq 0 \geq U(\bar{\xi}, \bar{\xi})$ implies that $\underline{\xi} \geq \bar{\xi}$. Recall the fact that $\underline{\xi}_{n+1} \leq \bar{\xi}_{n+1}$, which implies $\underline{\xi} \leq \bar{\xi}$. Hence it must be the case that $\underline{\xi} = \bar{\xi} = \xi_0^*$ where $y = \xi_0^*$ is the unique solution to $U(y, y) = 0$. Therefore, the only strategy that survives the iterated deletion of dominated strategies is the threshold strategy ξ_0^*. ■

Proof of Proposition B.2. Consider an investor who receives private signal x_i and believes that all other investors follow threshold strategy k. His expected payoff from staying in the bank and declining the intervention offer is

$$U_D(k, x_i) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x_i - \theta}{\sigma}\right) \pi\left(x_i, 1 - F\left(\frac{k - \theta}{\sigma}\right)\right) \mathrm{d}\theta.$$

The expected payoff from staying in the bank and accepting the offer is

$$U_A(k, x_i) = (1 - t) U_D(k, x_i) + s.$$

Therefore, the maximum expected payoff from investing is

$$U(k, x_i) = \max\{U_D(k, x_i), U_A(k, x_i)\}.$$

The following lemma will be useful later on.

Lemma 1 *Given that all other investors run on the bank if and only if their signal is below k, there exist two functions $k_1^*(k)$ and $k_2^*(k)$ such that an investor strictly prefers running if his private signal $x_i < k_1^*(k)$ and strictly prefers staying if $x_i > k_2^*(k)$. The functions $k_1^*(k)$ and $k_2^*(k)$ are given by*

$$k_1^*(k) = \inf\left\{k^* : U_D(k, k^*) \geq -\frac{s}{1-t}\right\},$$
$$k_2^*(k) = \sup\left\{k^* : U_D(k, k^*) \leq -\frac{s}{1-t}\right\}.$$

Both $k_1^*(k)$ and $k_2^*(k)$ are weakly increasing in k.

Proof of Lemma 1. The Lower Dominance Region assumption in Definition B.1 together with Limit Dominance (part (iii) of Assumption B.1) ensures that the functions $k_1^*(k)$ and $k_2^*(k)$ are well-defined. From the continuity of $U_D(k, x_i)$ in x_i it follows that

$$U_D(k, k_1^*(k)) = U_D(k, k_2^*(k)) = -\frac{s}{1-t}.$$

On the one hand, for any $x_i < k_1^*(k)$ we have $U_D(k, x_i) < -\frac{s}{1-t}$ and $U_A(k, x_i) = (1-t)U_D(k, x_i) + s < 0$. Therefore, $U(k, x_i) = \max\{U_D(k, x_i), U_A(k, x_i)\} < 0$ and so the agent will not invest if he observes $x_i < k_1^*(k)$. On the other hand, for any $x > k_2^*(k)$ we have $U_D(k, x_i) > -\frac{s}{1-t}$ and $U_A(k, x_i) = (1-t)U_D(k, x_i) + s > 0$. Hence $U(k, x_i) = \max\{U_D(k, x_i), U_A(k, x_i)\} > 0$ and so the agent will invest after observing signal $x_i > k_2^*(k)$.

Because $U_D(k, x_i)$ is weakly decreasing in k, it is straightforward to show that both $k_1^*(k)$ and $k_2^*(k)$ are weakly increasing in k. ■

Now we can use Lemma 1 to prove the uniqueness of equilibrium using iterated deletion of dominated strategies. Denote the run strategy by $a_i(x_i)$. We want to show that a strategy survives n rounds of iterated deletion of dominated strategies if and only if

$$a_i(x_i) = \begin{cases} 0 & \text{if } x_i < \underline{\xi}_n, \\ 1 & \text{if } x_i > \bar{\xi}_n, \end{cases}$$

where $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = \infty$. The terms $\underline{\xi}_n$ and $\bar{\xi}_n$ are defined inductively by $\underline{\xi}_{n+1} = k_1^*(\underline{\xi}_n)$ and $\bar{\xi}_{n+1} = k_2^*(\bar{\xi}_n)$.

Since $k^*(\xi)$ increases with ξ, it follows that $\underline{\xi}_n$ and $\bar{\xi}_n$ are (respectively) increasing and decreasing sequences. As $n \to \infty$, we have that $\underline{\xi}_n \to \underline{\xi}$ and $\bar{\xi}_n \to \bar{\xi}$; hence $\underline{\xi} = k_1^*(\underline{\xi})$ and $\bar{\xi} = k_2^*(\bar{\xi})$. Therefore, both $\underline{\xi}$ and $\bar{\xi}$ must be solutions to

$$U_D(\xi, \xi) = -\frac{s}{1-t}.$$

Let $l = 1 - F\left(\frac{\xi-\theta}{\sigma}\right)$ be the total funding staying in the bank when the fundamentals are θ and investors follow threshold strategy ξ; then the preceding equality can be rewritten as

$$\int_0^1 \pi(\xi, l)\,\mathrm{d}l = -\frac{s}{1-t}. \tag{C.18}$$

By Strict Laplacian State Monotonicity (part (ii) of Assumption B.1), the left-hand side of (C.18) is continuous and strictly increasing in ξ. Moreover, at the two boundaries, $\int_0^1 \pi(\underline{\theta}, l)\,\mathrm{d}l \le \pi(\underline{\theta}, 1) < -\frac{s}{1-t}$ and $\int_0^1 \pi(\bar{\theta}, l)\,\mathrm{d}l > 0 \ge -\frac{s}{1-t}$. Therefore, there exists a unique solution to (C.18): $\underline{\xi} = \bar{\xi} = \xi^*$. Note that, since ξ^* is independent of σ, it is (by iterated deletion of dominated strategies) the unique run threshold in equilibrium.

Given the run threshold ξ^*, we can solve for the private signal x_i such that $U_A(\xi^*, x_i) = U_D(\xi^*, x_i)$ or, equivalently, $U_D(\xi^*, x_i) = s/t$. It turns out that there exists a unique solution $x_i = \eta^*(\sigma)$ such that

$$U_D(\xi^*, \eta^*(\sigma)) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\eta^*(\sigma)-\theta}{\sigma}\right) \pi\left(\eta^*(\sigma), 1 - F\left(\frac{\xi^*-\theta}{\sigma}\right)\right) \mathrm{d}\theta = \frac{s}{t}. \tag{C.19}$$

The uniqueness of solution follows from (a) $U_D(\xi^*, x_i)$ increases with x_i, (b) $U_D(\xi^*, \xi^*) = -\frac{s}{1-t} < \frac{s}{t}$, and (c) $U_D(\xi^*, \xi^* + \sigma) = \pi(\xi^* + \sigma, 1) > \frac{s}{t}$. Since $U_D(\xi^*, x_i)$ increases with x_i, for any signal $x_i > \eta^*(\sigma)$, an agent strictly prefers investing and declining the participation offer.

Denote the limiting participation threshold $\lim_{\sigma\to 0} \eta^*(\sigma) = \eta$. Since $U_D(\xi^*, x_i)$ increases with x_i and since $U_D(\xi^*, \xi^*) < \frac{s}{t} = U_D(\xi^*, \eta^*(\sigma))$, we have $\eta^*(\sigma) > \xi^*$. It is then immediate that $\eta \ge \xi^*$. Next, we prove $\eta = \xi^*$ by way of contradiction. Suppose that $\eta > \xi^*$; then, taking $\sigma \to 0$ in the left-hand side of (C.19), we have

$$\lim_{\sigma\to 0} \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\eta^*(\sigma)-\theta}{\sigma}\right) \pi\left(\eta^*(\sigma), 1 - F\left(\frac{\xi^*-\theta}{\sigma}\right)\right) \mathrm{d}\theta = \pi(\eta, 1) \ge \pi(\xi^*, 1) > \frac{s}{t},$$

which contradicts (C.19). Therefore, $\lim_{\sigma\to 0} \eta^*(\sigma) = \eta = \xi^*$. ■

Proof of Proposition B.3. For any target $\xi^* \in (\theta_0, \xi_0^*)$, we show that the following intervention programs (s, t)

$$\left(-\frac{\pi(\xi^*, 1)}{\int_0^1 \pi(\xi^*, l)\,\mathrm{d}l} + 1 \right)^{-1} < t < 1 \tag{C.20}$$

and

$$s = -(1-t) \int_0^1 \pi(\xi^*, l)\,\mathrm{d}l, \tag{C.21}$$

satisfy conditions (i)–(iii) in Definition B.1 for partial-participation programs. First, condition (i) follows directly from equation (C.21). Second, equation (C.20) can be written as $\pi(\xi^*, 1) > -\frac{1-t}{t} \int_0^1 \pi(\xi^*, l)\,\mathrm{d}l$. Substituting in equation (C.21), we have $\pi(\xi^*, 1) > \frac{s}{t}$, and condition (ii) is satisfied. Finally, as stated in Proposition B.3, $\pi(\underline{\theta}, 1) \leq \int_0^1 \pi(\theta_0, l)\,\mathrm{d}l$. The Strict Laplacian State Monotonicity (part (ii) of Assumption B.1) implies $\int_0^1 \pi(\theta_0, l)\,\mathrm{d}l < \int_0^1 \pi(\xi^*, l)\,\mathrm{d}l = -\frac{s}{1-t}$. Therefore, condition (iii) is also satisfied. ■